Appendix D

Some Portfolio Theory Math for Water Supply

Constant-Reliability-Benefit Unit Costs

The reliability and cost of different water-supply options can vary, making comparisons between different options difficult. To create a level playing field, the Pacific Institute developed a method for adjusting estimated unit costs of water-supply options (including conservation and end-use efficiency) so as to keep the reliability for all options the same. The method borrows and adapts tools from financial portfolio theory.[3] It leads to constant-reliability-benefit unit costs that provide a more fair comparison between supply options with different uncertainty characteristics.

Finding constant-reliability-benefit unit costs involves a two-step process. First, a planner must specify a constant-reliability-benefit standard. For example, the water planner might say that water supply (or conservation measures) must equal drought year demand 97.5% of the time. Mathematically, this means that the annual average of the supply portfolio, A(P), minus two times[4] the standard deviation (SD) of the supply portfolio, SD(P), must be equal to future (planned for) drought-year demand, D_F:

$$A(P) - 2SD(P) = D_F \qquad (1)$$

Other reliability standards can be chosen according to a table present in any statistics textbook that shows the percentage of time a random variable will be more than a chosen multiple of the standard deviation from the average. For example, specifying a "1" in Equation 1 rather than a "2" yields a reliability standard of about 84 percent. Stated differently, a normally distributed random variable will be less than the average minus one standard deviation about 16% of the time, or one in six years.

The average supply of a portfolio is the sum of the average supplies of each of its parts. In our example, one compares combinations of the existing supply, A(E), with a new supply, A(N):

$$A(P) = A(E) + A(N) \qquad (2)$$

where $A(X) = \frac{1}{n}\sum_{i=1}^{n} Q_{xi}$

n = Number of years of annual flow data

Q_{xi} = Annual flow in year i from Source x

The standard deviation of a portfolio of sources depends on the standard deviation and average of each source, the correlation between the sources, and the percentage of water from each source. The standard deviation of a portfolio is the square root of the variance of the portfolio.

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[3] This work was supported in part by the U.S. Bureau of Reclamation. See Wolff and Kasower (2006).

[4] Or if expressed with an additional significant figure, as is common in statistics textbooks, 1.96.

The appropriate formula (modified by the author from Tucker et al. 1994) when two sources are involved is:

$$SD(P) = \sqrt{W(E)^2 S(E)^2 + W(N)^2 S(N)^2 + 2W(E)W(N)Rho(E,N)S(E)S(N)} \qquad (3)$$

where $W(E) + W(N) = 1$

$$W(X) = \frac{A(X)}{A(P)}$$

$$S(X) = \frac{SD(X)}{A(X)}$$

Rho(E,N) is the correlation coefficient between E and N

Formulas for the standard deviation and correlation coefficient (Rho) are provided in any statistics textbook, and one can calculate these summary statistics using a spreadsheet program. Combining Equations 1, 2, and 3 yields:

$$\sqrt{\left(\frac{A(E)}{A(P)}\right)^2 S(E)^2 + \left(\frac{A(N)}{A(P)}\right)^2 S(N)^2 + 2\left(\frac{A(E)}{A(P)}\right)\left(\frac{A(N)}{A(P)}\right) Rho(E,N)S(E)S(N)} = \frac{A(P) - D_F}{2A(P)} \qquad (4)$$

where $A(P) = A(E) + A(N)$

If one knows the average existing supply, the standard deviations of the existing and new sources of supply, and the correlation coefficient between supplies, Equation 4 will contain only one unknown, A(N). This is the average new supply required to ensure that the chosen reliability standard (97.5% in this case)[5] will be achieved. A(N) can be found by assuming a value for A(N), seeing how close or far apart the left and right hand sides of the equation are, and iteratively adjusting the assumed value until the value of A(N) that solves the equation is found. Table D-1 presents the solutions found in the body of this report (new surface water supply, desalination, and outdoor water conservation). Finally, the constant-reliability-benefit unit price for each option differs from the average unit price for each option by the ratio of A(N)/D_N. When A(N) equals growth in drought demand (D_N)[6], as with desalination and similar options, the average unit price for that water supply option is also the constant-reliability-benefit unit price. When A(N) is greater than or less than D_N, as with the surface water and outdoor conservation examples in Table D-1, the constant-reliability-benefit unit price for each option is higher or lower than the average unit price for that option, respectively.

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[5] Replacing the "2" in the denominator on the right hand side with the appropriate value, as discussed above, yields the appropriate equation for other reliability standards.

[6] Recall that D_N = equals D_F-D_E.

Table D-1: Unit Cost Reliability Premiums Under Various Assumptions

Water Supply Options	Coefficient of Variance (SD/A)	Correlation of Supply Options (Rho(E,N))	A(N)
Surface Water	20%	1.0	3,333 AFY
Desalination	0%	0.0	2,000 AFY
Outdoor Water Conservation	10%	-1.0	1,667 AFY

Notes: AFY = acre-feet per year

D_F = future drought-year demand

Assumes coefficient of variance of the existing source of 10%; A(E)=10,000 AFY; D_F=10,000 AFY; reliability level of about 97.5 percent.

Mathematics of Blending When Water Quality Is Uncertain

As in the reliability mathematics, a two-step process is used to determine the appropriate blending of water supply sources needed to obtain a specified water-quality objective. First, a planner must specify a water-quality standard and probability of achieving that standard. For example, the planner might specify that water quality must be 500 parts per million (ppm) total dissolved solids (TDS) at least 99.5% of the time. Mathematically, this means that the average quality of the supply portfolio, A(QP), minus three times the standard deviation of the portfolio's quality, SD(QP), must equal the water quality target (500 ppm):

$$A(QP) - 3SD(QP) = 500 \qquad (5)$$

Other probabilities of achieving the target standard can be chosen using a table present in any statistics textbook that shows the percentage of time a random variable will be more than a chosen multiple of the standard deviation from the average. For example, a reliability standard of about 84% requires specifying a "1" in Equation 5 rather than a "3." Specifying a "0" rather than "3" would mean water quality will be worse than 500 ppm 50% of the time. In this case, blended quality is simply the arithmetic average of the quality of the water sources.

The average quality of a portfolio is the weighted sum of the average qualities of the blended water sources. In our example, only two sources are blended at a time:

$$A(QP) = W(1)A(Q1) + W(X)A(QX) \qquad (6)$$

where $W(1)$ = Percent of the portfolio from Source 1

$W(X)$ = Percent of the portfolio from Source X

X = Source 2 or 3

$W(1) + W(X) = 1$

$$A(Qy) = \frac{1}{n}\sum_{i=1}^{n} q_{yi}$$

$A(Qy) =$ Average quality of Source y

y = Source 1, 2, or 3

n = number of years of annual average quality data

q_{yi} = annual average quality in year i from Source y

The standard deviation of the quality of a portfolio of sources, SD(QP), depends on the standard deviation and average quality of each source, the correlation between the source qualities, and the percentage of water from each source. The standard deviation of a portfolio is the square root of the variance of the portfolio. The appropriate formula (modified by the author from Tucker et al. 1994) when two sources are involved is:

$$SD(QP) = \sqrt{W(1)^2 S(1)^2 + W(X)^2 S(X)^2 + 2W(1)W(X)Rho(1,X)S(1)S(X)} \qquad (7)$$

where $S(y) = \dfrac{SD(Qy)}{A(Qy)}$

SD(Qy) = Standard deviation of the quality of Source y

Rho(1,X) = correlation coefficient between

the quality of Source 1 and the quality of Source X

Formulas for the standard deviation (SD) and correlation coefficient (Rho) are provided in any statistics textbook and one can calculate these summary statistics using a spreadsheet program. Combining Equations 5, 6, and 7 yields:

$$\sqrt{(1-W(X))^2 S(1)^2 + W(X)^2 S(X)^2 + 2(1-W(X))W(X)Rho(1,X)S(1)S(X)} = \frac{A(QP)-500}{3A(QP)} \qquad (8)$$

where $A(QP) = (1-W(X))A(Q1) + W(X)A(QX)$

and $S(y) = \dfrac{SD(Qy)}{A(Qy)}$

As with the reliability example, there is only one unknown in Equation 8 if one knows the summary statistics related to water quality for the water-supply options (average quality, standard deviation of quality, and correlation coefficient between quality measures). The unknown is W(X), the fraction of the blend with Source 1 that must come from Source X in order to maintain 500 ppm or better 99.5% of the time. As before, one must solve for W(X) by iteration. One then finds the fraction of the blend from source 1 by subtracting W(X) from 1. The cost of each blend that satisfies the quality specification is the weighted average cost using these fractions.

References

Tucker, A.L., K.G. Becker, M.J. Isambabi, and J.P.Ogden. 1994. Contemporary Portfolio Theory and Risk Management. West Publishing Company. St. Paul, Minnesota.