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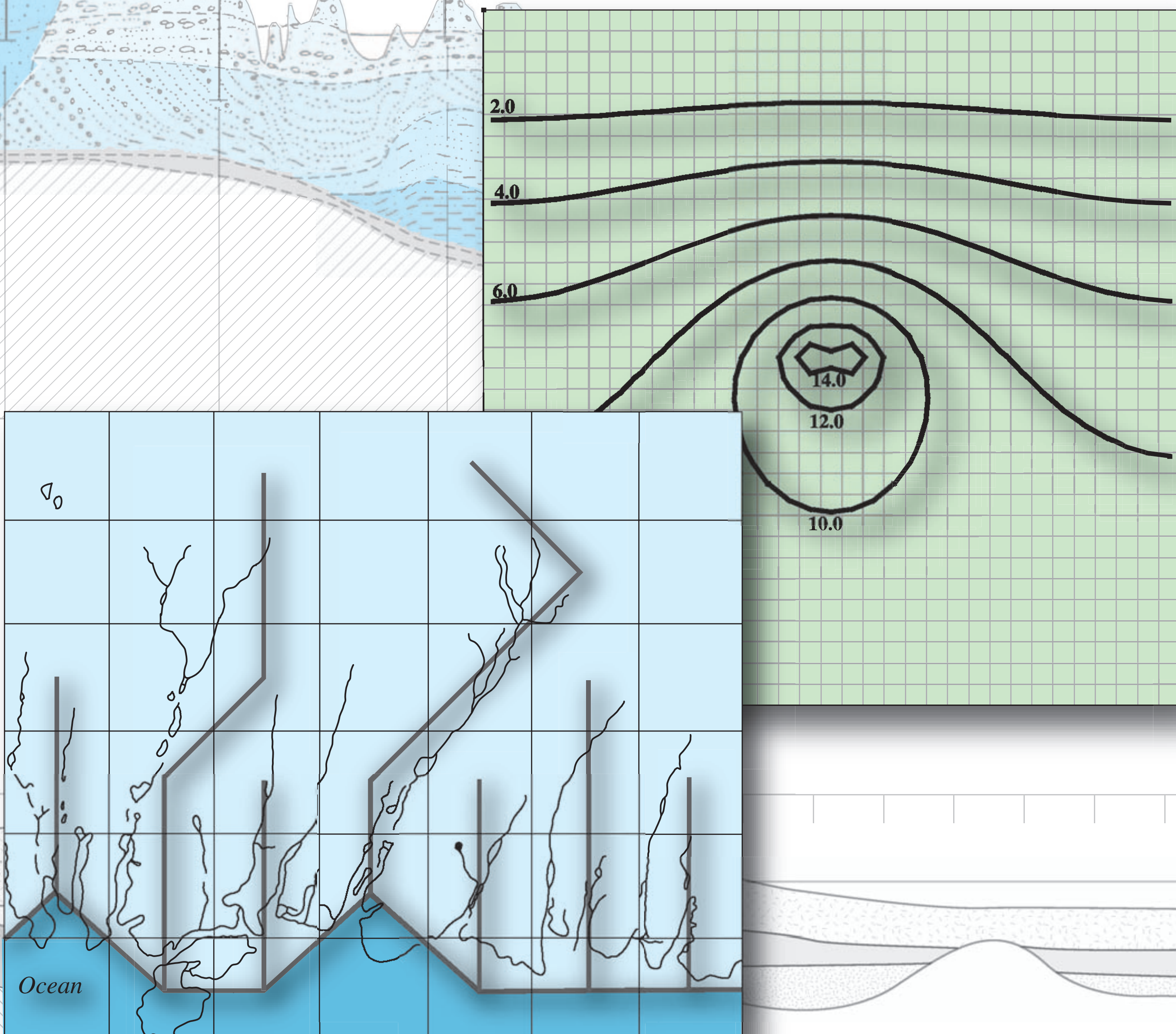
Guidelines for Evaluating Ground-Water Flow Models



Scientific Investigations Report 2004-5038

U.S. Department of the Interior
U.S. Geological Survey

SCWA-202

Guidelines for Evaluating Ground-Water Flow Models

By Thomas E. Reilly and Arlen W. Harbaugh

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Suggested citation:
Reilly, T.E., and Harbaugh, A.W., 2004, Guidelines for evaluating ground-water flow models: U.S. Geological Survey Scientific Investigations Report 2004-5038, 30 p.

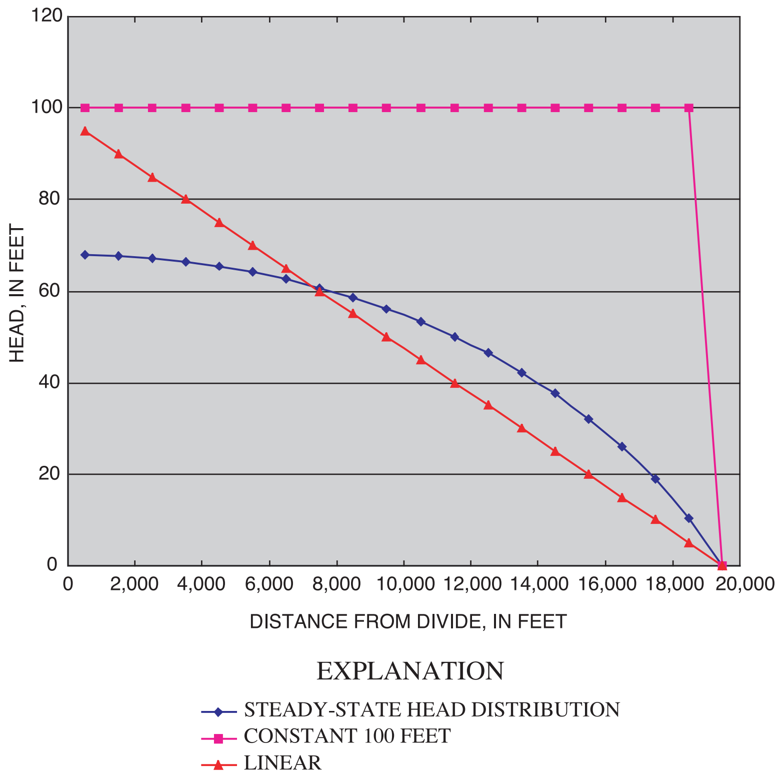

Figure 14. Head distribution along a model row from the divide to the constant-head node for three different initial conditions used for a transient simulation.

Accuracy of the Matrix Solution

Discrete numerical models involve the solution of large sets of simultaneous algebraic equations (Harbaugh and others, 2000). This solution of large sets of algebraic equations usually involves the use of sophisticated matrix solution techniques. Most of the solution techniques are iterative in nature whereby the solution is obtained through successive approximation, which is stopped when it is determined that a "good" solution has been obtained (Bennett, 1976). The criterion used in most iterative solution techniques is called the "head change criterion." When the maximum absolute value of head change from all nodes during an iteration is less than or equal to the selected head change criterion, then iteration stops.

When evaluating a ground-water flow model, even if the computer model has output results, one must check to determine if indeed a solution has been obtained by the matrix solution technique. The first check is to evaluate the head change criterion. Was the head change criterion set small enough to obtain a model solution with minimal error? One means of evaluating the head change criterion is to examine the global mass balance for the model. If the error in the mass balance (for example, total inflow minus total outflow divided by one half the sum of the inflow and outflow) over the entire model domain is small, usually less than

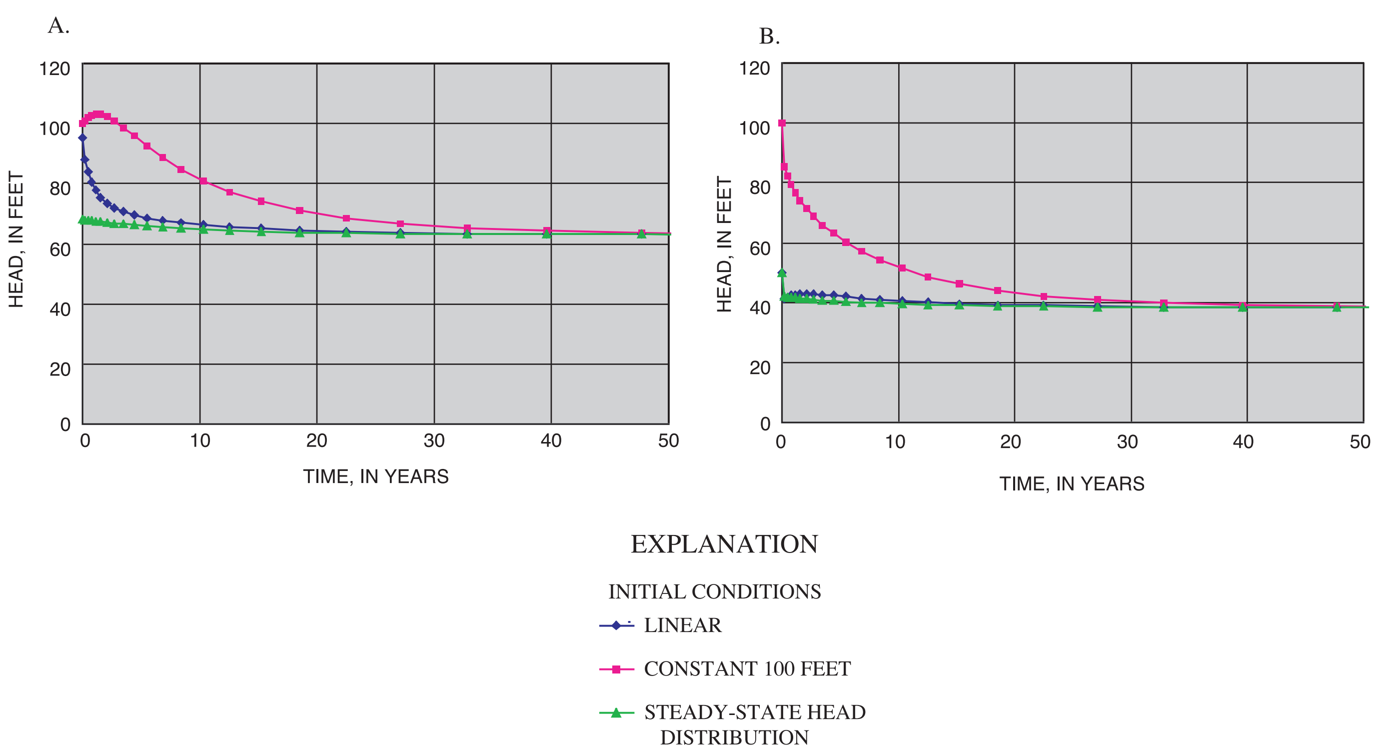

Figure 15. Head in a cell through time in response to a well discharging at a rate of 100,000 ft^3/d: (A) the head in layer 1 at the divide, and (B) the head in the cell with the discharging well in layer 3.

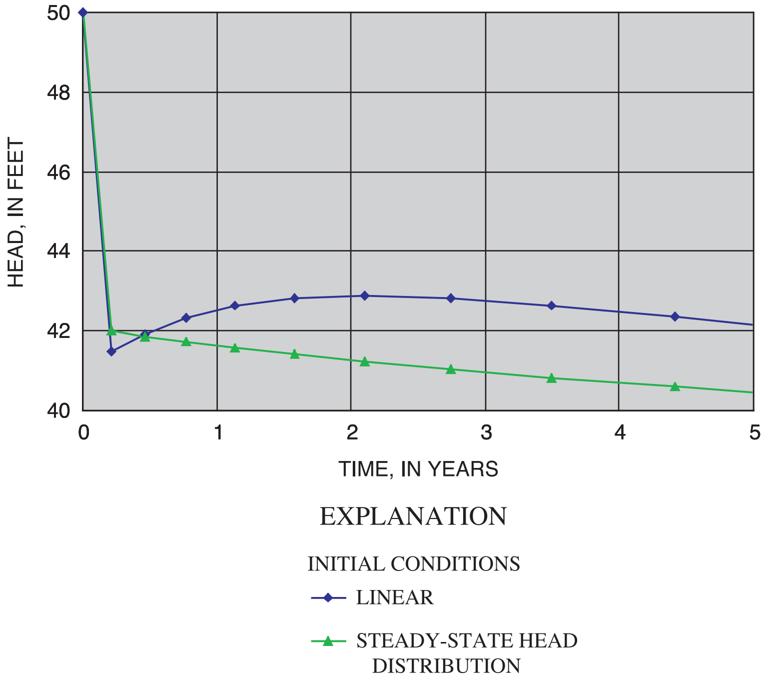

Figure 16. Head in the well for the first 5 years after the start of pumping for the cases using the initial conditions of the steady-state head distribution and the linearly varying head distribution.

0.5 percent, then the head change criterion is assumed to have been sufficient. If the error in the mass balance calculations is significant, then the matrix solution was not good and the model should be corrected by improving the matrix solution. The matrix solution can be improved by lowering the head change criterion, adjusting iteration parameters (if the solution techniques use iteration parameters), using different starting heads for steady-state simulations, or using a different solution technique.

Even if the head change criterion is met and the global mass balance error is small, the model solution may not be appropriate for the system under investigation. Two potential reasons are that some models can either be mathematically nonunique or very nonlinear. The mathematically nonunique problem usually is a poorly posed problem where a model has only specified-flow boundary conditions and no other boundary condition that specifies a head or datum (such as, constant head, river stage, general head boundary, etc.). In this type of problem, there is a family of solutions all with the same gradients but different absolute heads. The matrix solution technique may not converge or it may converge to one of the infinite number of possible solutions.

In nonlinear problems, the solution affects the coefficients of the matrix being solved; thus, the solution affects the problem being solved. As a result, the manner in which the iterative solution technique approaches a solution can affect the final solution. An example from Reilly (2001) illustrates this point. Consider a one-dimensional water-table system with a sloping impermeable bottom that contains a specified head and extends 5,000 m, with an areal recharge rate of 0.5 m/yr. The starting head for the equation solution is specified at 20 m, which is above all the bottom elevations of the cells but yet close to the magnitude of the expected results. Figure 17A is a cross-sectional view of a finite-difference representation of the steady-state solution. The cell farthest from the specified head is simulated as being dry. The total recharge flowing to the specified head cell for a 500-m width is 2,740 m^3/d. The convergence criterion of the model was met and the mass balance was excellent (showing 0.00 percent budget discrepancy). Now consider figure 17B, which is the result of a simulation of the same problem, except the starting head for the matrix solution was set at 100 m. As is shown in figure 17 and table 2, three cells are now simulated as being dry. The result is that less recharge is simulated as entering the model and the heads and water budgets are reduced accordingly, with only 2,055 m^3/d being represented as recharge entering the system for a 500-m width. Although both solutions converged and had excellent mass balances, at least one of them is incorrect. Because it is a nonlinear problem, it is not easy to determine which solution is correct. The rate of convergence and the method of making cells inactive must be considered and evaluated. After evaluating these aspects, and noting that the head in cell 7 (table 2 and fig. 17) of the second model is above the bottom elevation of cell 8, which was converted to dry during the iterative process, it seems that the first model most likely is correct. In the second model, the iterative solution, in attempting to converge, apparently overshot the bottom of some of the cells, which prematurely or erroneously truncated the area from the active model domain,

Table 2. Heads calculated for the same system with areal recharge and two different intitial heads.

[m, meters]

Cell number	Bottom elevation of cell	Head calculated with the initial head at 20 m	Head calculated with the initial head at 100 m
1	-30.0	0.00	0.00
2	-25.0	1.93	1.46
3	-20.0	3.83	2.86
4	-15.0	5.68	4.17
5	-10.0	7.49	5.38
6	-5.0	9.24	6.42
7	0.0	10.90	7.20
8	5.0	12.45	Dry
9	10.0	13.81	Dry
10	15.0	Dry	Dry