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An analytical framework for quantifying aquifer response time scales associated with transient boundary conditions

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SUMMARY

A major challenge in studying coupled groundwater and surface-water interactions arises from the considerable difference in the response time scales of groundwater and surface-water systems affected by external forcings. Although coupled models representing the interaction of groundwater and surface-water systems have been studied for over a century, most have focused on groundwater quantity or quality issues rather than response time. In this study, we present an analytical framework, based on the concept of mean action time (MAT), to estimate the time scale required for groundwater systems to respond to changes in surface-water conditions. MAT can be used to estimate the transient response time scale by analyzing the governing mathematical model. This framework does not require any form of transient solution (either numerical or analytical) to the governing equation, yet it provides a closed form mathematical relationship for the response time as a function of the aquifer geometry, boundary conditions, and flow parameters. Our analysis indicates that aquifer systems have three fundamental time scales: (i) a time scale that depends on the intrinsic properties of the aquifer, (ii) a time scale that depends on the intrinsic properties of the boundary condition, and (iii) a time scale that depends on the properties of the entire system. We discuss two practical scenarios where MAT estimates provide useful insights and we test the MAT predictions using new laboratory-scale experimental data sets.

1. Introduction

Understanding the interactions between groundwater and surface-water systems is an important aspect of water resources management. Using mathematical models to study these interactions can help us better address associated water quality and quantity issues. In the published literature, groundwater and surface-water interactions have been studied using both physical and mathematical approaches (Clement et al., 1994; Winter, 1995; Chang and Clement, 2012; Simpson et al., 2003a) that involve invoking a range modeling simplifications and assumptions, such as assuming that groundwater flow takes place in a homogeneous porous medium, assuming that streams are fully penetrating, and assuming rainfall conditions are uniform. To provide further insight into real-world practical problems, some of these simplifications and assumptions need to be relaxed.

A major challenge in studying groundwater and surface-water interactions arises from the fact that there is a considerable difference in the response times of these systems (Rodriguez et al., 2006; Hantush, 2005). For example, after a rainfall event, surface-water levels can respond on the order of hours to days, whereas groundwater levels might respond on the order of weeks to months. Current approaches for studying these problems can be classified into four categories, each of which involve certain limitations: (i) field investigations, which can be expensive and time consuming; (ii) laboratory experiments, which can be limited by scaling issues; (iii) numerical modeling, which, due to the orders of magnitude differences in the response times, might lead to numerical instabilities or other convergence issues (Hantush, 2005); and (iv) analytical modeling, which may be efficient but can have serious limitations in considering practical scenarios involving variations in stream stage, recharge, or discharge boundary conditions (Barlow and Moench, 1998). Several previous researchers have presented analytical solutions focussing on aquifer response times (Rowe, 1960; Pinder et al., 1969; Singh and Sagar, 1977; Lockington, 1997; Mishra and Jain, 1999; Ojha, 2000; Swamee and Singh, 2003; Srivastava, 2003).

Understanding groundwater response times near a groundwater surface-water boundary will help us make informed decisions about the use of different types of mathematical models. For

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example, if the water stage in the surface-water body is perturbed, we expect the local groundwater system in contact with the stream to undergo a transient response and eventually reach a new steady-state. Tools that can predict the time needed for such transient responses to relax to a steady-state condition could help to make informed decisions about using appropriate mathematical models. For example, immediately after changing the surface-water elevation, we would need to apply a transient mathematical model to predict the groundwater response; whereas, after a sufficiently long period of time, we could describe the system using a simpler steady-state model (Simpson et al., 2003b).

In the groundwater literature, *response time* (or lag time) is defined as the time scale required for a groundwater system to change from some initial condition to a new steady-state (Sophocleous, 2012). In the heat and mass transfer literature this time scale is known as the *critical time* (Hickson et al., 2009a,b, 2011). Simpson et al. (2013) summarized several previous attempts to estimate the groundwater response time into three categories: (i) numerical computation, (ii) laboratory-scale experimentation, and (iii) simple mathematical definitions or approximations. All three categories involve making subjective definitions of the response time by tracking transient responses and choosing an arbitrary tolerance ϵ and claiming that the response time is the time taken for the transient response to decay below this tolerance (Chang et al., 2011; Landman and McGuinness, 2000; Watson et al., 2010; Hickson et al., 2011; Lu and Werner, 2013). There are several limitations with this approach. The most obvious limitation is that the response time depends on a subjectively defined tolerance, ϵ. Secondly, this approach does not lead to a general mathematical expression to describe how the response time would vary with problem geometry, changes in boundary conditions or aquifer parameters. Finally, this approach requires an analytical or a numerical solution to the governing transient equation. To deal with these limitations, Simpson et al. (2013) demonstrated the use of a novel concept, mean action time (MAT), for estimating aquifer response times.

The concept of MAT was originally proposed by McNabb and Wake (1991) to describe the response times of heat transfer processes. MAT provides an objective definition for quantifying response time scales of different processes (McNabb, 1993). MAT analysis leads to an expression relating the response time to the various model parameters. Simpson et al. (2013) used MAT to characterize the response time for a groundwater flow problem that was driven by areal recharge processes, but did not consider any groundwater and surface-water interactions. The objective of this study is to extend the work of Simpson et al. (2013) and present a mathematical model which describes transient groundwater flow processes near a groundwater and surface-water boundary with time-dependent boundary conditions. We adapt existing MAT theory to deal with time-dependent boundary conditions and present expressions for MAT which describe spatial variations in response times for both linear and non-linear boundary forcing conditions. These theoretical developments are then tested using data sets obtained from laboratory experiments.

2. Mathematical development

We consider a one-dimensional, unconfined, Dupuit–Forchheimer model of saturated groundwater flow through a homogeneous porous medium (Bear, 1979), which can be written as,

$$S_y \frac{\partial h}{\partial t} = K \frac{\partial}{\partial x}\left[h \frac{\partial h}{\partial x}\right], \tag{1}$$

where $h(x,t)$ (L) is the groundwater head at position x, t (T) is time, S_y (–) is the specific yield and K (L/T) is the saturated hydraulic conductivity. When variations in the saturated thickness are small compared to the average saturated thickness, we can linearize the governing equation by introducing an average saturated thickness, $\bar{h}$, to yield (Bear, 1979),

$$S_y \frac{\partial h}{\partial t} = K\bar{h}\frac{\partial^2 h}{\partial x^2}, \tag{2}$$

which can be re-written as the linear diffusion equation,

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}, \tag{3}$$

where $D = K\bar{h}/S_y$ (L^2/T) is the aquifer diffusivity. In this work, we will use Eq. (3) to model a groundwater system which changes from an initial condition, $h(x,0) = h_0(x)$, to some steady-state, $\lim_{t\to\infty} h(x,t) = h_\infty(x)$. We will consider two different classes of boundary conditions for Eq. (3): Case 1, in which both the left ($x = 0$) and right ($x = L$) boundary conditions vary as functions of time, and Case 2, in which one boundary condition is fixed and the other one is allowed to vary with time.

2.1. Case 1: two time varying boundary conditions

We first consider the case where the surface-water variations at both the left ($x = 0$) and right ($x = L$) boundaries vary with time,

$$B_L(t) = h(0,t), \tag{4}$$

$$B_R(t) = h(L,t). \tag{5}$$

We assume that, after a sufficient amount of time, both $B_L(t)$ and $B_R(t)$ approach some steady condition,

$$\lim_{t\to\infty} B_L(t) = h_\infty(0), \tag{6}$$

$$\lim_{t\to\infty} B_R(t) = h_\infty(L), \tag{7}$$

for which the steady solution of Eq. (3) is,

$$h_\infty(x) = \left(\frac{h_\infty(L) - h_\infty(0)}{L}\right)x + h_\infty(0). \tag{8}$$

A schematic of these initial, transient and steady-state conditions are shown in Fig. 1.

The purpose of this study is to present an objective framework to estimate the time scale required for the system to effectively relax to steady-state conditions. To begin our analysis we first consider the following two mathematical quantities (Ellery et al., 2012a,b; Simpson et al., 2013),

$$F(t;x) = 1 - \left[\frac{h(x,t) - h_\infty(x)}{h_0(x) - h_\infty(x)}\right], \quad t \geqslant 0, \tag{9}$$

$$f(t;x) = \frac{dF(t;x)}{dt} = -\frac{\partial}{\partial t}\left[\frac{h(x,t) - h_\infty(x)}{h_0(x) - h_\infty(x)}\right], \quad t \geqslant 0, \tag{10}$$

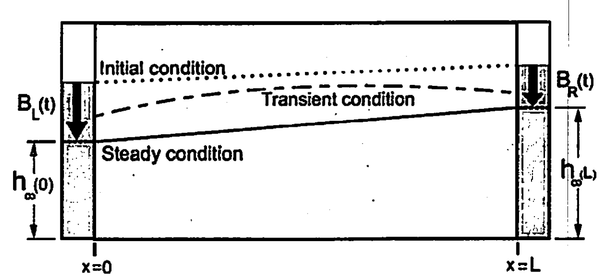

Fig. 1. Schematic of the physical model showing initial (dotted), transient (dashed) and steady (solid) conditions. Changes in water head in the right and left boundaries are defined by functions of $B_R(t)$ and $B_L(t)$, respectively. At steady-state, the left and right boundary conditions reach the levels $h_\infty(0)$ and $h_\infty(L)$, respectively.

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