Dynamic optimization of crop production with nonuniform irrigation and nitrogen carryover and leaching is considered. A production function system with thresholds, plateau maximum, and yield reduction is estimated from experimental data; rapid convergence to a steady-state is observed. Spatial variability implies a 40% increase in applied water and a six-fold increase in nitrate emissions, while dynamic optimization has more modest impacts. Nitrate emission control is accomplished primarily through reduced applied water, illustrating a strong cross-policy effect. Significant levels of water conservation and nitrate pollution control are achieved at relatively low cost with traditional irrigation systems and baseline conditions.

**Key words:** dynamic optimization, nitrogen management, nonuniform irrigation, water management.

Irrigated agriculture constitutes approximately 70% of global freshwater consumption. While the nearly 260 million hectares of irrigated land worldwide currently provide 40% of the global food supply, future expansion and intensification is likely necessary to meet a predicted 40% to 45% increase in food demand by the year 2025 (United Nations Environment Program 1999), implying additional stress on a scarce natural resource. Irrigated agriculture is also a major source of groundwater nitrate pollution. Violations in the maximum allowable levels of nitrates in drinking water are reported in every European country, while African nitrate loads in some suburban groundwater wells are six to eight times World Health Organization acceptable levels. A survey of nearly 200,000 U.S. water sampling records found that more than 2 million people drank water exceeding federal nitrate standards, and nearly 52% of the community water wells and 57% of the domestic wells are considered nitrate contaminated (Nolan et al. 1998). In California, nitrates are responsible for more well closures than any other chemical, and 10% to 15% of the water supply wells violate federal standards (Bianchi and Harter 2002).

In response to the potential health threats from nitrates in groundwater, a variety of regulations on irrigated agriculture have been proposed and implemented, including limits on fertilizer usage and nitrate concentrations in groundwater (Shortle and Abler 2001). Research addressing the groundwater nitrate problem has often focused on policies targeting the nitrogen input (e.g., Choi and Feinerman 1995; Nkonya and Featherstone 2000); understandably so given that annual fertilizer use, which has been estimated to add 7 billion pounds more nitrogen than is taken up by the plants on the field, has increased since the 1990s (National Research Council 1993; USDA 2005). Yet as highlighted in research by Helfand and House (1995) and Larson, Helfand, and House (1996) in a static field-level analysis of lettuce production, the complementarity between applied water rates and nitrate pollution is such that a second-best approach consisting of a water surcharge is only marginally less efficient than an emissions charge, albeit substantially more efficient than...
a nitrogen input charge. An added benefit from a water surcharge is a reduction in applied water, an underpriced, oversubsidized resource subject to a substantial literature of its own (Caswell, Lichtenberg, and Zilberman 1990). The complementarity between water, a scarce natural resource, and nitrates, an environmental quality problem, demonstrates the need to consider water and nutrient management policies jointly, as stressed in Lee (1998), and the potential for cross-policy effects, as shown in Weinberg and Kling (1996) for water markets and drainage policy.

Dynamic analysis of water and nitrogen inputs, crop yield, and nitrate emissions is quite limited (Segarra et al. 1989; Vickner et al. 1998). Furthermore, an issue of long-standing concern in the agronomic, soil science, and agricultural engineering literatures is field-level spatial variability in soil and irrigation system parameters (Nielsen, Biggar, and Erh 1973; Seginer 1978). While this variability has several consequences, the main implication is that irrigation water is typically distributed nonuniformly over a field with consequent impact on water infiltration, soil/plant processes, crop yields, deep percolation flows, and nitrogen leaching. Although this topic has seen only modest attention by agricultural economists, it is invariably critical when considered. In particular, Berck and Helfand (1990) show that von-Liebig-type production functions at the plant-level integrate to smooth nonlinear functions at the field level. Feinerman, Letey, and Vaux (1983) show theoretically that spatial variability typically increases profit-maximizing applied water rates, while Letey, Vaux, and Feinerman (1984) demonstrate that optimum water applications under spatial variability can differ by factors of two or more compared to uniform applications and more closely correspond to observed behavior. Similarly, Dinar, Letey, and Knapp (1985) establish that field-level spatial variability is critical to accurately analyzing salinity and drainage problems associated with irrigated agriculture. Finally, Larson, Helfand, and House (1996) express caution in policy instrument choice without more research on the variance of nitrate leaching due to field-level heterogeneity, while Chiao and Gillingham (1989) incorporate nonuniformity for applied phosphorous in dry land production.

Within the water-nitrogen economics literature, the only study to incorporate dynamic spatial variability is Vickner et al. (1998) with spatial variability defined as the fraction of a field under- or overirrigated relative to a water requirement. They conclude that ignoring irrigation application variability understates nitrate abatement policies. Their model of nonuniform irrigation differs from models typical of the irrigation economics literature, and results in some 95% of land area uniformly overirrigated and hence represented by a single parameter.3 Somewhat contrary to Helfand and House (1995) and Larson, Helfand, and House (1996), they find that nitrogen control is a preferable second-best strategy to controlling applied water. Further analysis of this problem therefore seems crucial to natural resource usage and the environment in irrigated agriculture.4

This article further explores spatial heterogeneity, dynamic optimization, and nitrate emissions in irrigated agriculture with attention toward water-nitrogen complementarity and possible cross-policy effects. A spatial dynamic model of water and nitrogen management is developed with endogenous water and nitrogen applications and interseasonal nitrogen carryover. This model extends the irrigation and nitrogen economics literature by characterizing water infiltration with a spatial density function over the field. A major task is estimation of a plant-level model for yield, carryover, and emissions, where the function must exhibit appropriate global properties to account for water infiltration above and below mean levels. To this end, data from an unusually rich field trial are used to estimate a production function system exhibiting thresholds, plateau maximums, and input substitution.

Fundamental properties of the dynamic system are investigated, including decision rules, spatial moments, and evolution of the soil nitrogen spatial density function. A key finding is rapid convergence to a steady-state under a wide variety of initial conditions, which

3 Nitrogen dynamics in Vickner et al. (1998) also differs from the dynamics modeled in this article. They specify nitrogen carryovers for both the under- and over-irrigated portions of the field; however, these fractions are endogenous and can vary over time. The carryover equations are, therefore, inaccurate if portions of the field in a given year are a mix of previous fractions. While not likely quantitatively significant in their application, this could be a difficulty elsewhere. The equations of motion in the model developed here are for exogenous fractions of the field and avoid this difficulty.

4 A sophisticated literature on precision agriculture exists for rain-fed agriculture. While irrigated producers can do little to mitigate infiltration variability for a given irrigation system, they could in principle modify fertilizer applications as a reviewer pointed out. The scientific and engineering information to analyze this does not exist for irrigated agriculture to our knowledge. Regardless, not all spatial variability can be met, and analyses as here are necessary for benefit/cost calculations of precision farming activities.
is significant for regional policy analysis as it simplifies needed computations and data. Specification tests are conducted for spatial variability and dynamic optimization; consistent with previous literature, spatial variability is fundamental for water scarcity and environmental quality degradation in irrigated agriculture. The effects of a range of water and nitrogen emission prices are evaluated also. There is a significant policy-relevant response from water and nitrogen management alone, even while crop and irrigation system are fixed. As in Johnson, Adams, and Perry (1991), the implication is that significant resource conservation and environmental quality improvement is possible at relatively low cost to agricultural productivity, at least starting from current conditions. The results also exhibit large cross-policy effects complementing Larson, Helfand, and House (1996) and Weinberg and Kling (1996).

Bioeconomics of Field-Scale Crop Growth and Management

Spatial dynamics of field-level water and nitrogen management is analyzed. Water is distributed nonuniformly over the field in response to soil heterogeneity and/or nonuniform irrigation systems, implying spatially variable water uptake and nitrogen uptake and emissions. Interseasonal carryover dynamics for soil nitrogen are also considered. With variability in the various driving factors, soil nitrogen also exhibits heterogeneity over time even with initial soil nitrogen uniformity. Field-scale crop yield and emissions in each period are an integration over the field; hence, spatially variable water infiltration directly impacts current crop yield and nitrogen emissions, and indirectly affects future levels by inducing soil nitrogen variability. The importance of spatial variability and dynamics in water and nitrogen management is analyzed along with input pricing policies for water conservation and water quality at the field level.

Letting $r$ denote the discount rate and $T$ the planning horizon, the present value of net benefits to land and management ($$/ha) is

$$\pi = \sum_{t=0}^{T} \left[ p_y \bar{y}_t - p_w \bar{w}_t - p_n \bar{n}_{at} - \kappa - p_e \bar{n}_{at} \right] (1 + r)^{-t}$$

where $t$ is time [years], $\bar{y}_t = \text{field-scale crop yield} [\text{Mg/ha}]$, $\bar{w}_t = \text{field-average applied water depth} [\text{cm}]$, $\bar{n}_{at} = \text{applied nitrogen} [\text{kg/ha}]$, and $\bar{n}_{et} = \text{nitrogen emissions/leaching} [\text{kg/ha}]$. Parameters are $p_y$, $p_w$, and $p_n$ as the prices of crop [$$/Mg], water [$$/ha/cm], and nitrogen [$$/kg], respectively; $\kappa$ is nonwater and nonnitrogen production costs associated with the cropping system [$$/ha], and $p_e$ is nitrogen leaching cost [$$/kg].

Spatial variability is a dynamic extension of the static model proposed by SEGNER (1978) and used subsequently in Feinerman, Letey, and Vaux (1983), Dinar, Letey, and Knapp (1985), and Berck and Helfand (1990). The key concept is a water infiltration coefficient giving the fraction of field-average water depth infiltrating at a point in the field. At a particular point in the field, the amount of water infiltrating into the root zone at time $t$ is $w_t(\beta) = \beta \bar{w}_t$, where $\beta \in [0, \infty]$ is the water infiltration coefficient. $\beta$ is distributed according to a spatial density function, $f(\beta)$, with mean $E[\beta] = 1$, and standard deviation $\text{SD}[\beta]$ that depends on the type of irrigation system.

Field-level relationships for yield and nitrogen emissions are:

$$\bar{y}_t = \int_0^\infty y_t(\beta) f(\beta) \, d\beta$$

$$\bar{n}_{et} = \int_0^\infty n_{et}(\beta) f(\beta) \, d\beta$$

where $y_t(\beta)$ and $n_{et}(\beta)$ are plant-level yield [$\text{Mg/ha}$] and nitrogen emissions [$\text{kg/ha}$], respectively. Thus field-level crop yield and nitrogen emissions are plant-level yield and emissions integrated over the field according to the spatial density function for water infiltration. Plant-level production functions for yield and nitrogen emissions are $y_t(\beta) = g_y[n_t(\beta), w_t(\beta), n_{at}(\beta)]$ and $n_{et}(\beta) = g_e[n_t(\beta), w_t(\beta), n_{at}(\beta)]$, respectively, where $n_t$ is inorganic soil nitrogen [$\text{kg/ha}$] at the beginning of period $t$, and $n_{at}$ is applied nitrogen. At the plant-level, crop yield and nitrogen emissions, specified as leaching below the rootzone, depend on initial soil nitrogen, water infiltration, and nitrogen applications at points in the field characterized by $\beta$.

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5 All irrigation systems exhibit nonuniform water distribution. This includes travel and residence time disparities (furrow), friction losses (sprinkler and drip), and emitter variability (drip and LEPA). Well-maintained modern systems can achieve significant infiltration uniformity with higher yield and/or reduced water inputs. This article focuses on furrow systems but the model applies to investment in any system.
Soil nitrogen dynamics or carryover dynamics (Segarra et al. 1989) for a given $\beta$ are

$$n_{t+1}(\beta) = g_n[n_t(\beta), w_t(\beta), n_{at}(\beta)]$$

indicating dependence on the same variables as plant-level crop yield and nitrogen emissions. Initial soil nitrogen in period 1 is assumed constant across the field $[n_1(\beta) = \bar{n}_1]$, and nitrogen is applied uniformly across the field $[n_{at}(\beta) = \bar{n}_{at}]$, the latter assumption following from the use of mechanical/chemical fertilizer applications consistent with irrigated agriculture. The model can be modified to make plant-level fertilizer applications proportional to infiltrated irrigation water; however, this is not pursued here. For computational tractability in the dynamic optimization model, the spatial density support is discretized into a series of intervals, each with a specified $\beta$ value and representing a fraction of the field as computed from the spatial density function. A useful interpretation is that the field is divided into a finite number of plots each with a specified $\beta$ value and area.\(^6\)

Control variables are field-level applied water $w_t$, and nitrogen $n_{at}$, and state variables are nitrogen carryover for each of the discrete grid intervals for the $\beta$ infiltration coefficients. The dynamic optimization problem is solved using the GAMS CONOPT nonlinear optimization procedure. To eliminate endpoint effects, the optimization routine is implemented as a running horizon problem in which a sequence of finite-horizon optimization problems are solved with a thirty-year time horizon, each starting from the states resulting from the first period of the previous solution and retaining only the first period results from each for the final solution.

### Economic Data and Crop-Water-Nitrogen Production Function

The empirical application is corn production in Yolo County, California with a traditional (furrow one-half mile) irrigation system. Maximum corn yield is 12.02 Mg/ha, with a price of $102.02 [Mg^{-1}]$. Production costs include costs such as seed, land preparation, and machinery but do not include those associated with water, nitrogen fertilizer, land and management, and cash overhead (UCCE 2004). Irrigation system data are from University of California Committee of Consultants (UCCC 1988). Combined, amortized nonwater production costs are $673 ha^{-1}$, baseline nitrogen fertilizer costs are $0.59 kg^{-1}$, and baseline water costs are $0.64 [ha cm]^{-1}$. We assume a discount rate of 5% with all economic data inflation-adjusted to 2003 dollars.

The infiltration coefficients $\beta$ are distributed lognormally over the field with $E[\beta] = 1$ for mass balance. The baseline results assume a Christensen Uniformity Coefficient (CUC) of 0.77, where CUC is a widely used measure of nonuniformity in the irrigation engineering literature, and calculated as $1 - \int_{0}^{\infty} |\beta - 1| f(\beta) d\beta$. SD[\beta] was estimated so that the CUC = 0.77 under the lognormal $\beta$ distribution. This distribution is discretized into 11 possible $\beta$ values, each with an associated fraction of the field computed as $\int_{\beta}^{\beta+\Delta\beta} f(\beta) d\beta$, where $f$ is the lognormal density and $\Delta\beta = 1.11$, is a partition of $[0, \infty]$ containing the discrete $\beta$ values. This model can be interpreted as 11 subareas of the field, each characterized by a $\beta$ value, constituting a specified fraction of the field, and with an associated soil nitrogen state variable.

A classic work on water-nitrogen production functions is Hexum and Heady (1978). Although they investigate several functional forms, they settle on polynomials (including fractional powers) as a useful functional form. Ackello-Ogutu, Paris, and Williams (1985), among others, point out that polynomials generally do not fit qualitative agronomic theory and evidence: they have a point maximum instead of a plateau maximum, allow more substitution than is warranted by the data, and imply excessive input usage. Moreover, von-Liebig functions demonstrate superior data fit relative to polynomials and other traditional smooth production functions (Ackello-Ogutu, Paris, and Williams 1985; Grimm, Paris, and Williams 1987; Paris 1992). However, a recent sophisticated statistical analysis by Berck, Geoghegan, and Stohs (2000) rejects both the von-Liebig formulation as well as the non-substitution hypothesis. Taken together, these results leave open the appropriate form for plant-level production functions.

\(^6\) We consider the downward movement of water and nutrients in the rootzone only and no horizontal interaction within the rootzone, implying that only the distribution of infiltration coefficients is necessary. The particular spatial configuration of the field is not needed and, in general, there are infinite spatial configurations consistent with an assumed distribution. The assumed sub-areas of the field with a given $\beta$ value need not be contiguous. Also note that this formulation still implies externalities. Nitrates percolate below the rootzone to the water table and then move laterally through various mechanisms, eventually influencing water quality throughout the aquifer.
Figure 1. Corn plant-level production function. Corn yield $y_t$, nitrogen emissions $n_{et}$, and nitrogen carryovers $n_{t+1}$ as functions of applied water $w_t$ and applied nitrogen $n_{at}$ for initial soil nitrogen $n_t = 160$ kg/ha in year $t$.

Additional concerns arise at the field-level with spatial variability. As outlined in Lanzer and Paris (1981; figure 1), a general conceptual model of yield production functions exhibits convex-concave behavior initially, followed by a yield plateau and then possibly a yield decline. In the uniform case, only the concave portion is economically relevant, hence functional forms constituting local approximations (e.g., Taylor series approximations via polynomials) may be reasonable as the optimization model can appropriately bound the inputs. In the spatial case, though, some parts of the field likely receive input levels in the convex (increasing returns to scale) portion, while other parts receive excess input levels leading to yield declines. Consequently, functions with desirable global properties and data fit are necessary, raising additional issues to those debated in the literature. Polynomials with any reasonable order and von-Liebig functions are unlikely to perform well globally even if they are reasonable locally.

To overcome some of these difficulties, we develop a production function system specified by several component functions representing the major flows and processes in the plant-water-soil system. One reason for the system approach rather than the approach used in much of the literature (e.g., Johnson, Adams, and Perry 1991; Vickner et al. 1998) is that a system approach can capture yield-depressing effects associated with excess water infiltration in a logical fashion while still allowing individual component functions to be estimated with classical properties. We also utilize functional forms that exhibit convex-concave behavior and plateau maximums. These functional forms effectively place upper and lower bounds on the levels for individual variables. In combination with multiplicative functions such as Mitscherlich-Baule (Paris 1992), this system allows for input substitution consistent with Berck, Geoghegan, and Stohs (2000), yet subject to limits consistent with the classic findings of Paris and others.

The plant-level production system was estimated for corn using an unusually rich data set from Tanji et al. (1979) (see also Pang, Letey, and Wu 1997a, b). The experimental data consist of two years of corn field trials at a University of California-Davis site from October 1974 through September 1976. The trials measure the effects of nitrogen and water applications rates on yields, nitrogen uptake, inorganic soil nitrogen levels, nitrate emissions, and organic nitrogen mineralization. The experiment provides data beyond that typically used in the agricultural production economics literature (e.g., Hexum and Heady 1978 as used in Berck, Geoghegan, and Stohs 2000), and is key to the analysis as it allows estimation and testing of the system model without resorting to hidden variables and speculative functional forms. It should be emphasized that while these field experiments were performed in the mid 1970s, recent articles in the soil science
literature still calibrate to this data (Pang, Letey, and Wu 1997a,b).

Details of the estimated system are in the Appendix. The estimated functions fit the data extremely well ($R^2 \geq 0.78$) and have appropriate global properties. The composite plant-level production functions for yield, emissions, and carryover as functions of soil nitrogen, applied water, and applied nitrogen are constructed from this system and illustrated in figure 1. While generally consistent with prior irrigation economics research, the results can differ with respect to nitrogen and water interactions. In figure 1a, for example, excessive water application rates at low soil nitrogen levels decrease yields as the additional water leaches nitrogen out of the soil. As more nitrogen is leached out of the soil with excessive water application (figure 1b), less is then available as carryover into the next period (figure 1c). Knapp and Schwabe (2007) contain additional discussion and graphs of the estimated production function system.

**Dynamics of the Spatially Variable Field**

With spatially variable water infiltration and nitrogen carryover dynamics, the field constitutes a relatively complex dynamic system. In this section, computational experiments are used to characterize the dynamic system with the base water price and a zero nitrogen emissions price. Figure 2 partially characterizes the optimal decision rule giving applied water and nitrogen as a function of soil nitrogen. In the figure, soil nitrogen is constant across the field; thus this is only a partial characterization as soil nitrogen in each of the grid cells can take on a range of nonuniform values in principle. As illustrated, water applications are reasonably constant across the range of values; applied nitrogen generally declines linearly to a threshold, after which it is zero.

Time series of the spatial means of the state and control variables were computed starting from (uniform) initial soil nitrogen $n_{i0}$ of 50 kg/ha and 350 kg/ha. The time paths

![Figure 2. Optimal decision rules for applied water and nitrogen by initial soil nitrogen](image-url)
converge to a steady-state independent of the initial conditions, and convergence is rapid (<10 years) even with initial conditions relatively far from the steady-state. This property was found in all of the empirical specifications reported in the article, and similar rapid convergence has been reported in the salinity economics literature (Dinar and Knapp 1986; Knapp 1992; Letey and Knapp 1995). While growers in an actual operating environment need to evaluate and respond to initial conditions in the field, the significant implication for modeling and policy analysis is that one can reasonably focus on the optimal steady-state, thereby lessening the data and computational burden at farm and regional spatial scales. This is important as it would be virtually impossible to estimate and solve full dynamic systems for all fields at larger spatial scales.

Temporal evolution of the spatial density function for soil nitrogen provides a more detailed view. A piece-wise linear cumulative distribution function (CDF) for soil nitrogen in each year was computed from model output on soil nitrogen state variables and their associated fractional areas. This CDF shows the fraction of the field having soil nitrogen levels less than or equal to a specified value; spatial density functions are estimated from the CDF by finite-differences over an appropriate grid. Figure 3 depicts the spatial density function for soil nitrogen for several years. Consistent with the results for first moments, this spatial density function is relatively invariant after approximately eight years indicating a steady-state for the entire system. Note that the observed rapid convergence to a limiting density function does not imply that the problem can be modeled as a static system; the equations of motion are necessary to compute the steady-state while formal dynamic optimization procedures are required to compute the dynamically optimal steady-state.8

Current corn prices are substantially higher than the price assumed in our analysis, a price change largely due to recent increases in the demand for corn to produce ethanol. Knapp and Schwabe (2007) provide a sensitively analysis over a range of corn prices.

what dependent on the selected grid interval for soil nitrogen values, an issue that generally arises with any nonparametric density estimation. A grid with 11 intervals was selected here as being most informative. The grid interval for the estimated density function is independent and conceptually distinct from the number of state variables. At any point in time, 0, 1, or multiple state variables could take values lying within a specified nitrogen interval in figure 3. The discretization determining the number of plots in the field and state variables is for β values; the fact that there are the same number of intervals in figure 3 as there are state variables is purely coincidental.

8 Additional spatial results are presented in Knapp and Schwabe (2007). Referring to figure 1(c), results show that infiltrated water occurs in the convex, concave, and plateau maximum of the emissions function, supporting the earlier conceptual discussion that global plant-level functions are needed with spatial variability. Another figure demonstrates that the bulk of N-emissions in the steady-state come from plots with intermediate β values; low β values imply low deep percolation depths hence reduced N leaching; high β values imply low soil N levels entering the year and hence reduced N available to be leached.
Table 1. Optimal Steady-State Values Under Alternative Discount Rates

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>Applied Water</th>
<th>Applied Soil Nitrogen</th>
<th>Nitrogen Emissions</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>r (%)</td>
<td>w_{ss} (cm)</td>
<td>n_{a,ss} (kg/ha)</td>
<td>n_{e,ss} (kg/ha)</td>
<td>y_{ss} (tons/ha)</td>
</tr>
<tr>
<td>0</td>
<td>87.5</td>
<td>224</td>
<td>161.4</td>
<td>10.08</td>
</tr>
<tr>
<td>5</td>
<td>87.9</td>
<td>221</td>
<td>158.9</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>88.2</td>
<td>219</td>
<td>156.8</td>
<td>35.8</td>
</tr>
<tr>
<td>15</td>
<td>88.5</td>
<td>217</td>
<td>155.2</td>
<td>35.7</td>
</tr>
<tr>
<td>20</td>
<td>88.7</td>
<td>215</td>
<td>153.8</td>
<td>35.6</td>
</tr>
</tbody>
</table>

Note: PV-optimization with spatial variability and base parameter values as specified in the text. Variables of the form \( x_{ss} \) denote steady-state values.

Table 2. Optimal Steady-State Values by Behavioral Regime (PP versus PV) and Spatial Heterogeneity (Uniform versus Spatial)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>PV-Spatial</td>
<td>88</td>
<td>221</td>
<td>159</td>
<td>36</td>
<td>231</td>
<td>10.08</td>
</tr>
<tr>
<td>PP-Spatial</td>
<td>91</td>
<td>190</td>
<td>138</td>
<td>33</td>
<td>213</td>
<td>9.9</td>
</tr>
<tr>
<td>PV-Uniform</td>
<td>63</td>
<td>208</td>
<td>173</td>
<td>6</td>
<td>237</td>
<td>10.1</td>
</tr>
<tr>
<td>PP-Uniform</td>
<td>65</td>
<td>171</td>
<td>137</td>
<td>7.3</td>
<td>212</td>
<td>9.8</td>
</tr>
</tbody>
</table>

Note: Behavioral regimes are Present Value Optimization (PV) and Period-by-Period Optimization (PP). Discount rate = 5% and baseline parameter values, including \( p_w = \$0.64/(ha-cm) \) and \( p_e = 0 \).

Holding other prices fixed, a 50% increase in corn price to the current market rate of approximately $153/Mg results in over a 60% increase in nitrogen emissions. Such a large potential increase in nitrogen emissions can exacerbate greatly an already existing nitrogen pollution problem. However, other factors, such as the price of nitrogen, which is surging lately as well due to fuel price increases, will likely regulate some of the behavioral response to the ethanol-generated price changes.

A key parameter in any dynamic analysis is the discount rate. Risk-free interest rates and rates of return on agricultural and general assets typically are fairly low (4% to 5%) in developed countries. Sustainability concerns, however, have spawned a large literature on discounting as an appropriate criterion in view of intergenerational equity over long horizons. At the other end of the spectrum, capital scarcity in developing countries can imply larger discount rates. Table 1 explores alternate discount rates and optimal management. Higher discount rates tend to increase water applications and decrease nitrogen applications slightly resulting in reduced nitrogen supply and crop yields. These results are consistent with the observation that increased discount rates imply reduced concern for the future. The results also suggest declining nitrogen emissions with increased interest rates. In general, though, the quantitative changes are modest and indicate relatively little sensitivity to discount rates.

Spatial Variability and Dynamic Optimization: Specification Tests

This section evaluates the significance of spatial variability and dynamic optimization. That is, do analyses require consideration of these factors, or can they safely be neglected? As before, base prices, a 5% interest rate, and a zero nitrogen emission price (\( p_e = 0 \)) are considered. Table 2 contrasts steady-state values for water and nitrogen management with and without spatial variability under alternate assumptions on optimal behavior, either present value (PV) or period-by-period (PP) optimization. PP optimization selects input levels in each time period to maximize profits in that period conditioned on the states entering that period; states for the next period are calculated from the equations of motion and selected input levels. In contrast to PV optimization, the impacts of current decisions on future periods are ignored under PP optimization.

Introducing spatial variability can have very significant impacts. As shown in table 2,
applied nitrogen rates \( (n_s) \) increase by a modest 6% under PV-Spatial relative to PV-Uniform. However, consistent with previous literature, optimal steady-state applied water rates increase substantially by nearly 40%. Feinerman, Letey, and Vaux (1983) demonstrate that the latter effect arises because, after a threshold level, spatial variability increases the marginal product of water resulting in increased optimal applied water. Crop yields are not affected by uniformity in the water-nitrogen empirical example here; however, profits are somewhat lower under nonuniform irrigation reflecting the increased costs of greater inputs.

The most striking effect is on nitrate emission rates. As table 2 demonstrates, a level of spatial variability associated with a traditional irrigation system results in optimal steady-state nitrate emissions six times greater than that predicted in the uniform scenario. This is primarily due to the applied water effect noted above, that is, the desire to maintain adequate moisture levels in all parts of the field results in over-irrigating some parts of the field; consequently, higher levels of deep percolation result with subsequent increases in nitrate emissions. Such results also explain the higher applied nitrogen levels as compensation for the reduced soil nitrogen levels.

The implications of these observations for agricultural natural resource and environmental policy are compelling. As noted above, computed water application rates are much closer to observed values than those from uniform calculations (e.g., those reported in Hexum and Heady (1978)). More novel here, the results also raise the hypothesis that observed nutrient loadings to environmental media may be much more due to field-level spatial variability than to lack of emission prices. Certainly it would not be possible to understand current nitrate loadings from irrigated agriculture—and by extension grower policy response—without accounting for spatial variability. Similar results can be expected for other agricultural chemicals in irrigated agriculture as well.

We also conducted sensitivity analysis on the infiltration coefficient to further test this hypothesis. The baseline results for this article assume a CUC = 0.77; however, CUC values for a specific field depend on a variety of factors including soil type and variability, management and upkeep activities, weather conditions, and so on. Our value is likely at the high end for furrow systems: UCCC (1988) report a CUC for a traditional furrow system of 0.70 but also note—based on experimental evidence—that this value may be high. Model results are \( \{w_{ss}, n_d, n_e, \pi_{ss}\} = \{117 \text{ cm}, 216 \text{ kg/ha}, 44 \text{ kg/ha}, \$159/\text{ha}\} \) for CUC = 0.70 and \( \{123 \text{ cm}, 214 \text{ kg/ha}, 42 \text{ kg/ha}, \$152/\text{ha}\} \) for CUC = 0.65. Thus as the CUC decreases from a favorable estimate of 0.77 to an average estimate of 0.70 (i.e., a decrease in the infiltration uniformity), applied water rates increase by 33%, applied nitrogen rates decrease by 2%, nitrogen emissions decrease by 22%, and annual net benefits decrease by 5%. These results strengthen the baseline findings that field-level spatial variability is a major determinant of input decisions, crop yield, and emissions.

A related question that arises is if it is possible to approximate this system with a simpler representation. We will not explore this topic in detail due to space limitations, but several comments can be made. The analysis in Vickner et al. (1998) divides the field into adequately watered and inadequately watered portions, although their empirical results end with 95% of the field being adequately watered, in essence a single homogenous cell in the model. As previously noted, they found a two-fold difference in emission rates due to spatial variability; in contrast, we find a sixfold difference. Although the empirical settings are different, the difference in findings, at least in part, may be due to the differences in representing spatial variability.9

Focusing on the nonuniform case (PV-Spatial versus PP-Spatial), PV optimization results in slightly reduced water application levels (3%) relative to PP optimization, but significantly increased nitrogen application levels (17%). These results follow from the introduction of forward-looking decisions in the PV framework.10 Shadow values of soil nitrogen are positive provided soil nitrogen is not at an excessive or nonproductive level. Applied water leaches nitrogen and hence reduces carryover (i.e., interseasonal) nitrogen, while applied nitrogen increases carryover levels. It follows that introducing positive shadow values into the decision calculus for a given year

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9 Analyses can implicitly include spatial variability via an irrigation efficiency coefficient (Caswell, Litchenberg, and Zilberman 1990). However, the plant-level production functions used here are highly nonlinear, and integration over even LRP production functions implies nonlinear field-level production functions (Berk and Helfand 1990). This approach appears as a linear approximation and a priori could only be expected to hold over a limited range of input values. Johnson, Adams, and Perry (1991) include subfields in a crop-simulator model.

10 Kennedy (1986) and Segarra et al. (1989) evaluate dynamic optimization versus annual optimization for nitrogen application, but not for irrigation or spatial variability.
will induce the observed input differences in table 2. Of course, this result could be reversed if initial soil N is sufficiently high to have a low marginal product (MP) for crop yield, high MP for emissions, and emission damages are large enough.

The higher soil and applied nitrogen levels under PV optimization combine to compensate for the lower applied water rates resulting in higher crop yields and annual net benefits relative to the PP optimization. Although not strikingly large at first glance, the $5/ha difference is $4800/yr for a 388-hectare operation, a much more significant sum. This per-unit area difference is relatively stable across a range of water prices and emission charges. Sensitivity analysis for nitrogen input prices also was conducted to reflect energy costs and circumstances where nitrogen inputs are charged to reduce nitrate emissions. Prices ranged from 10% to 50% over the baseline value. Similar to Segarra et al.’s (1989) spatially homogenous model, higher nitrogen prices increased the value of optimal management by a relatively small amount (3% to 5%).

The effect of dynamic optimization on emission rates is less predictable as decreased water inputs and increased nitrogen inputs have opposing impacts on emissions. As shown in table 2, PV optimization results in slightly higher emissions (9%). Similar qualitative effects were found for a range of nitrogen input prices. However, other results (not reported) show that the dynamic optimization specification reduces emission rates for both higher water prices and nitrogen emission prices. In some cases (e.g., $p_e = $0.20/kg) the ancillary reduction in emissions from PV optimization is significant (24%). Consistent with intuition, the qualitative, incidental effect of introducing full dynamic optimization on emissions is ambiguous but possibly quite significant.

### Water Conservation

Irrigation water is underpriced due to subsidies, externalities, lack of markets, and average-cost pricing. These factors, plus anticipated population and economic growth and environmental concerns, suggest higher prices and/or transfers to other economic sectors and the environment. A survey of the literature was conducted to identify potential water prices and transfers relevant to California agriculture (Knapp and Schwabe 2007). The results of this survey suggest water price increases of 50% (or more) and/or quantity reductions of 10% to 20% are possible for the average California agricultural water user over several decades. Table 3 analyzes the sensitivity of the optimal steady-state to irrigation water price using the dynamic optimization model. The results in table 3 demonstrate an inelastic own-price derived demand for water management. In particular, a 10% change in water prices induces minimal changes in applied water, while a 20% change induces a 17% reduction in applied water.

Although derived water demand is inelastic, the response is potentially significant from a policy perspective because in many regions

### Table 3. Water Price Effects on Optimal Steady-State Values

<table>
<thead>
<tr>
<th>Water Price $p_w$ ($/ha-cm$)</th>
<th>Applied Water $\bar{w}_{st}$ (cm)</th>
<th>Applied Nitrogen $\bar{n}_{a,ss}$ (kg/ha)</th>
<th>Soil Nitrogen $\bar{n}_{s,t}$ (kg/ha)</th>
<th>Yield $\bar{y}_{st}$ (tons/ha)</th>
<th>Nitrogen Emissions $\bar{n}_{e,ss}$ (kg/ha)</th>
<th>WTP for Water $\pi_w$ ($/ha$)</th>
<th>Grower Profit $\pi_p$ ($/ha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64 (Current/Baseline)</td>
<td>88</td>
<td>221</td>
<td>158.8</td>
<td>10.1</td>
<td>36</td>
<td>227.01</td>
<td>168.27</td>
</tr>
<tr>
<td>0.71 (10% Increase)</td>
<td>86.1</td>
<td>221.1</td>
<td>160.7</td>
<td>10.1</td>
<td>34.9</td>
<td>226.95</td>
<td>166.34</td>
</tr>
<tr>
<td>0.77 (20% Increase)</td>
<td>73.4</td>
<td>216.8</td>
<td>162.2</td>
<td>9.94</td>
<td>25.2</td>
<td>213.17</td>
<td>156.80</td>
</tr>
<tr>
<td>0.83 (30% Increase)</td>
<td>72.3</td>
<td>216.2</td>
<td>169.1</td>
<td>9.93</td>
<td>24.0</td>
<td>212.50</td>
<td>152.34</td>
</tr>
<tr>
<td>0.90 (40% Increase)</td>
<td>71.4</td>
<td>215.7</td>
<td>169.9</td>
<td>9.92</td>
<td>23.0</td>
<td>211.78</td>
<td>147.81</td>
</tr>
<tr>
<td>0.96 (50% Increase)</td>
<td>70.5</td>
<td>215.2</td>
<td>170.6</td>
<td>9.91</td>
<td>22.2</td>
<td>211.05</td>
<td>143.37</td>
</tr>
</tbody>
</table>

Note: PV-optimization with spatial variability and other parameter values at baseline values as specified in the text. $WTP$ for Water = revenue less costs not including water charge. Grower Profit = Annual NB less water charge.
agriculture is the dominant water user. In California, where agriculture uses approximately 80% of the statewide withdrawals, a 17% reduction in water use—if universally applicable—expands urban water use by 67%. Note also that the cost of this released agricultural water is quite low; for the 20% water price increase, the average cost of released water as measured by the net loss in agricultural production is $0.95/ha-cm ($11.69/ac-ft), well below the marginal values in residential and industrial uses, or the marginal cost of new supply development. Associated yield effects are very minimal, implying that current water prices are inducing water use with fairly low marginal productivity.

Equally striking, and perhaps surprising, is the effect of water price increases on nitrogen emissions. With the 20% water price increase, the subsequent reduction in water applications reduces deep percolation sufficiently to cut nitrate emissions by 30%, a result primarily driven by water management as applied nitrogen only falls by 2%. An unanticipated result of this research is therefore that there can be very significant cross-policy effects. An understanding of this result follows from the fundamental considerations noted earlier: nitrogen leaching in irrigated agriculture is driven by water flows below the rootzone, and these can be quite significant due to spatial variability. This result indicates that nitrogen management (and by extension other nutrients and agri-chemicals) is in a very real sense water management.

Crop and irrigation systems are emphasized in economic research on irrigated agriculture; the results here suggest considerable potential for water and nutrient management though there are limitations. Only a traditional irrigation system is considered; modern irrigation systems are typically more uniform and this could limit management response. The analysis is conducted for corn because of data availability although corn is not the major crop in California. This is not necessarily limiting as corn water use rates are comparable to, and in some instances less than, use by other major crops such as cotton (61–107 cm) and tomatoes (107 cm). Water costs vary in California and water management potential may already have been exploited in high-cost areas. While a full analysis of water and nutrient management for natural resources and the environment is considerably beyond the scope of this study, the results suggest at a minimum the need for enhanced research on this topic.

## Nitrate Emission Control

Irrigated agriculture is one of the largest polluting sectors of the economy due to its physical scale and the difficulties attendant to nonpoint regulation. Nitrate emissions from irrigated agriculture lead to a variety of damages, including human health and eutrophication with consequent ecological impacts. The above analyses demonstrate that field-scale spatial variability is a major contributor to nitrate loadings. This raises the question as to whether agricultural productivity can be maintained while reducing pollutant loadings.

Table 4 describes optimal steady-state management as dependent on a charge (marginal damages) on nitrogen emissions ranging from $0.20 to $1. As the charge increases, emissions are reduced by reductions in applied water and nitrogen. While soil nitrogen increases as a consequence of these input reductions, both yield and annual net benefits decline. Notably, water applications show the greatest response to changes in the emissions charge. For instance, to achieve a 58% reduction in nitrate emissions, water applications are reduced by 29%, applied nitrogen only 8%. Accordingly, the most efficient approach to minimizing the impacts of the emissions charge is to reduce

<table>
<thead>
<tr>
<th>N Emissions Charge $p_e$ ($/kg$)</th>
<th>Applied Water $\bar{w}_{ss}$ (cm)</th>
<th>Applied Nitrogen $\bar{n}_{a,ss}$ (kg/ha)</th>
<th>Soil Nitrogen $\bar{n}_{s,ss}$ (kg/ha)</th>
<th>Yield $y_{ss}$ (tons/ha)</th>
<th>Nitrogen Emissions $\bar{n}_{e,ss}$ (kg/ha)</th>
<th>Annual NB $\bar{NB}$ ($/ha$)</th>
<th>Grower Profit $\bar{P}$ ($/ha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>88</td>
<td>221</td>
<td>158.8</td>
<td>10.1</td>
<td>36</td>
<td>168.27</td>
<td>168.27</td>
</tr>
<tr>
<td>0.20</td>
<td>72</td>
<td>213.2</td>
<td>166.7</td>
<td>9.908</td>
<td>23.5</td>
<td>166.59</td>
<td>161.89</td>
</tr>
<tr>
<td>0.40</td>
<td>69.7</td>
<td>209.7</td>
<td>166.8</td>
<td>9.866</td>
<td>20.9</td>
<td>165.83</td>
<td>157.47</td>
</tr>
<tr>
<td>0.60</td>
<td>68</td>
<td>207.1</td>
<td>166.7</td>
<td>9.831</td>
<td>19.1</td>
<td>164.87</td>
<td>153.41</td>
</tr>
<tr>
<td>0.80</td>
<td>66.2</td>
<td>205</td>
<td>166.4</td>
<td>9.8</td>
<td>17.4</td>
<td>164.09</td>
<td>150.17</td>
</tr>
<tr>
<td>1.00</td>
<td>63.5</td>
<td>203</td>
<td>168.2</td>
<td>9.75</td>
<td>15.2</td>
<td>161.89</td>
<td>146.69</td>
</tr>
</tbody>
</table>

Note: The emission charge $p_e = 0$ represents the baseline case. Annual $NB = $ revenue less costs not including $N$ charge. Grower $Profit = Annual NB less $N$ charge.
applied water rates by a greater percentage than applied nitrogen rates, the effect being more nitrogen remaining on the field and less leaching through the soil.

These results demonstrate substantial nitrogen emission reduction with minimal impact on agricultural productivity or social net benefits. For an emissions price of $1, emissions are reduced by 58%, while yield and social net benefits decline by 3% and 13%, respectively. While additional control will eventually become increasingly expensive, these results are broadly consistent with the findings for other pollutants in which substantial reductions from uncontrolled levels can be achieved at relatively low costs (Tietenberg 2006). These results again demonstrate rather large and possibly surprising cross-policy effects, namely that nitrate emission pricing engenders a large drop in irrigation water, consistent with the earlier hypothesis that field-scale spatial variability is a major determinant of pollutant loadings.

The emissions charge induces efficient management for the associated level of N-emissions, and if the charge equals the marginal damages, then full social efficiency is achieved. Emission reductions also can be induced by input-side instruments. As an example, Tables 3 and 4 show that a water price of $0.83/ha-cm leads to almost identical results as the nitrogen emissions charge of $0.20/kg. In general, efficient input-side policy requires charges on all pollution generating inputs (Griffin and Bromley 1982), implying a surcharge for both water and nitrogen applications. The efficient input charges can be computed using the shadow values associated with the equations of motion, but this is not pursued here due to space limitations. A closely related topic is equity effects on grower profits, which generally depend on the selected policy instruments. For instance, in the example given the emission charge is somewhat more favorable to the grower than the water charge. Again, we do not pursue this topic in detail as a full analysis needs to account for rebates or tiered pricing that influence equity even for a given choice of instruments, as well as entry/exit considerations.

Conclusions

The article develops a spatial dynamic optimization model of field-scale water and nitrogen management. The model incorporates spatial variability consistent with the agro-economic and irrigation engineering literature, includes nitrogen carryover dynamics, and estimates a plant-level production function system exhibiting substitution consistent with Berck, Geoghegan, and Stohs (2000) while subject to limits as implied by Paris (1992). Qualitative dynamics exhibited by the model indicate a relatively rapid convergence to the optimal steady-state independent of initial conditions. This finding has potentially significant implications for quantitative policy analysis. If dynamics and optimization are important and transition time-scales long, then accurate regional policy analysis requires specifying initial conditions for all fields and solving a very large optimization problem, a heroic task from a data and computational perspective. The results here suggest that the essentials of the problem are well-captured by the dynamically optimal steady-state, a computationally and informationally much more tractable problem.11

Spatial variability is fundamental to resource scarcity and environmental quality in irrigated agriculture. While spatial variability does not imply large changes in nitrogen applications, it does have very large effects on water applications and nitrogen emissions such that overlooking spatial variability leads to erroneous results. The results demonstrate that input demand, pollutant loadings, and grower response are much larger than would be predicted from a uniform model. The extent to which simplifications used in the agricultural production economics literature are an acceptable approximations, and over what range, is an open question requiring further investigation. The model developed here can be used as tested for this purpose.

Dynamic optimization versus static (period-by-period) optimization also was tested. Static optimization implies lower nitrogen application rates and higher water application rates than PV-optimality. Higher water applications leach additional nitrogen out of the soil leaving less carryover for future periods and, consequently, less nitrogen uptake and lower yields. While the quantitative loss from static optimization is not large in percentage terms, it can

11 Even just the optimal steady-state in this model is likely still too complex for direct inclusion in a regional programming model with many crops, irrigation systems, and land quality types. As an alternative, this field-level model with a given crop and irrigation system can be run over a range of water, nitrogen, and emission prices, and a regression model fit to the resulting optimal steady-state values for applied water, applied nitrogen, crop yield, and nitrogen emissions. These estimated production functions can be included in the regional programming model; this would be a computationally feasible system for a large number of activities.
still translate into significant farm-level losses. Emission effects, meanwhile, are ambiguous as the static optimization procedure reduces applied nitrogen but increases applied water.

Water conservation and nitrate pollution control policies are evaluated as well. While estimated water demand is inelastic, water price increases well within estimated values consequent to a variety of possible policy reforms can result in policy-relevant quantity reductions. For example, a 20% water price increase from the base level here still leaves the price considerably less than the true shadow value facing California agriculture as calculated in other studies; this price increase, though, induces water reductions, which if scaled to all of California, would imply almost a two-thirds increase for urban uses. In the quantity dimension and given the crop and water prices considered, establishing a needed 10% to 20% agricultural water transfer rate to support urban growth and environmental restoration goals in California over the next several decades can be achieved with an annual loss of $15/ha or less in agricultural net benefits. Of course, equity consequences for growers would depend on specific policy mechanisms and instruments.

Similar findings hold for nitrate pollution control. The results suggest that efficient emission reductions are achieved primarily through reduced applied water relative to nitrogen fertilizer, a direct result of spatial variability. As with water, and starting from baseline conditions, significant reductions in nitrate emissions are obtained with relatively modest consequences for agricultural production. In particular, a $1/kg emission charge that induces a 55% emissions reduction incurs only a 6% loss in agricultural net benefits. This result holds starting from no regulation and for the crop and water prices considered here. Eventually, though, nitrate regulation becomes increasingly expensive as standards are tightened. Note that the water and nitrate results follow from crop management solely; irrigation systems and crop choice as stressed in previous work are not considered. Consideration of these strategies strengthens the results as additional compliance methods further reduce the already low costs found here.

An unanticipated finding of this research is a very strong cross-policy effect: water management implies strong reductions in nitrogen emissions, while emissions management implies large reductions in applied water. These results follow from the observation that nitrogen is transported through the rootzone via water flows, and the latter are larger than might be anticipated to maintain adequate moisture levels in all portions of the field. This complements Weinberg and Kling (1996) who find strong cross-policy effects for regional water and drainage management, and Larson, Helfand, and House (1996) who illustrate, both theoretically and empirically, the complementary relationship of water and nitrate pollution. Interestingly, the results differ from Vickner et al. (1998) who find that nitrogen management is more efficient than water management implying lower cross-over effects on water conservation.

The findings in this article suggest that nitrogen management in irrigated agriculture is as much water management as it is nitrogen input policy. In particular, the role of field-scale water infiltration variability appears crucial; it does not seem possible to either understand existing levels of resource demand/environmental loadings, or to accurately model and predict growers’ policy response, without consideration of this phenomena. It can be readily hypothesized that this is likely the case for other nutrients and agri-chemicals in irrigated agriculture as well.

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References


Appendix

Plant-Level Production Function System

The plant-level production function system consists of six component functions representing the major soil/plant processes and fluxes. After estimation, this system specifies composite functions giving crop yield, nitrogen emissions, and carryover dynamics as functions of initial (inorganic) soil nitrogen and applied water and nitrogen at a point within the field as characterized by \( \beta \). Integration over \( \beta \) then determines field-scale production relations.

Corn yield \( y_t \) with maximum potential yield \( y_{\text{max}} \) [Mg/ha] is

\[
y_t(\beta) = y_{\text{max}} \left( \frac{1}{1 + \left( \frac{25 + w_t(\beta)}{w_{50}} \right)^{-\phi_{w}}} \right) \\
\times \left( 1 + \frac{1}{3} \left( \frac{n_{\text{tr}}(\beta)}{n_{\text{max}}(\beta)} \right)^{-\phi_{n}} \right)
\]

where, \( w_t(\beta) \) is infiltrated water [cm], \( n_{\text{tr}}(\beta) \) is plant nitrogen uptake [kg/ha], and \( w_{50} \) and \( n_{\text{max}} \) are scaling coefficients for infiltrated water and nitrogen uptake implying 50% and 75% maximum crop yields, respectively (these allow parsimonious function estimation and representation). The parameters to be estimated are \( y_{\text{max}}, w_{50}, \phi_{w}, n_{\text{tr}}, \phi_{n}, \) and \( n_{\text{max}} \). In equation (A.1), crop yield is convex-concave in the individual inputs with a plateau maximum at \( y_{\text{max}} \); the multiplicative form allows a degree of input substitution.

Plant nitrogen uptake \( n_{\text{tr}}(\beta) \) with maximum potential plant uptake \( n_{\text{max}} \) is

\[
n_{\text{tr}}(\beta) = n_{\text{tr}}^{\max} \left( \frac{1}{1 + \left( \frac{25 + w_t(\beta)}{w_{50}} \right)^{-\phi_{w}}} \right) \\
\times \left( 1 + \frac{1}{3} \left( \frac{n_{\text{tr}}(\beta)}{n_{\text{max}}(\beta)} \right)^{-\phi_{n}} \right)
\]

where \( n_{\text{tr}}(\beta) \) is nitrogen supply [kg/ha], \( w_{50} \) and \( n_{\text{max}} \) are scaling parameters, and the estimated parameters are \( w_{50}, \phi_{w}, n_{\text{tr}}, \phi_{n}, \) and \( n_{\text{max}} \). Equation (A.2) has similar qualitative characteristics as the yield function (A.1). Nitrogen supply is defined by the accounting relation

\[
n_{\text{tr}}(\beta) = n_t(\beta) + n_{\text{tr}}(\beta) - n_{\text{le}}(\beta)
\]

where \( n_t(\beta) \) is inorganic soil nitrogen at the beginning of the season [kg/ha], \( n_{\text{tr}}(\beta) \) is applied nitrogen [kg/ha], and \( n_{\text{le}}(\beta) \) is nitrogen leaching from the soil [kg/ha].

Equation (A.4) specifies nitrogen leaching as a function of initial soil nitrogen, along with applied nitrogen and infiltrated water.
(A.4) \[ n_{ez}(\beta) = \frac{\phi_4 n(\beta) + n_{at}(\beta)}{1 + e^{-\phi_{tw}w(\beta) - w_{50}}} \]

where \( \phi_4, w_{50}, \) and \( \phi_{tw} \) are parameters to be estimated. In this relation, nitrogen emissions are a fraction of soil nitrogen supply. This fraction is zero for low levels of infiltrated water, consistent with minimal transport below the rootzone due to low soil moisture levels, but increases in a convex-concave manner, eventually approaching a value of one as infiltrated water depths become large enough. Thus the maximum amount that can be leached is the measure of nitrogen supply consistent with mass balance. The parameter \( w_{50} \) is a scaling parameter as above.

Inorganic soil nitrogen loss from denitrification, volatilization, and other factors, \( n_{ez}(\beta)[\text{kg/ha}] \), is defined as

(A.5) \[ n_{ez}(\beta) = \phi_{20} + \phi_{21}[n_{at}(\beta) + n_{l}(\beta)] + \phi_{22}[n_{at}(\beta) + n_{l}(\beta)]^2 + \phi_{tw} w_{l}(\beta) \]

where \( \phi_{20}, \phi_{21}, \phi_{22}, \) and \( \phi_{tw} \) are fitting parameters to be estimated. In general, inorganic nitrogen losses \( n_{ez} \) depend on soil nitrogen supply but can also be influenced by water supply. Finally, with these definitions we can specify soil inorganic nitrogen dynamics as

(A.6) \[ n_{r+1}(\beta) = n_r(\beta) + \bar{n}_{at} - n_{at}(\beta) - n_{ez}(\beta) - n_{r}(\beta) \]

which is an accounting identity reflecting mass balance. In particular, ending soil inorganic nitrogen equals initial soil inorganic nitrogen plus applied nitrogen minus inorganic nitrogen losses to uptake, leaching, and denitrification and other factors.

These relations were estimated with data from Tanji et al. (1979) and Pang et al. (1997a), which provide values for all variables. One observation for \( n_{ez} \) in (A.5) was theoretically implausible and inconsistent with other observed values. This observation was treated as an outlier and replaced with a value determined by extrapolation. Graphical analysis and a trial and error specification search identified functional forms with suitable data fit and global properties. Estimated coefficients and associated statistics are reported in Table A1. The estimated regressions provide excellent fit with \( R^2 \) values ranging from 0.78 to 0.95, and all estimated parameter values are significant at the 95% level or higher. Graphical analysis of the regressions indicate functional fits lying within bands defined by data in alternate years, and all exhibit global properties consistent with the generalized conceptual model in Lanzer and Paris (1981).