WATER RESOURCES RESEARCH, VOL. 23, NO. 9, PAGES 1738–1750, SEPTEMBER 1987

Stochastic Model for the Long-Term Transport of Stored Sediment in a River Channel

HARVEY M. KELSEY

Department of Geology, Western Washington University, Bellingham

ROLAND LAMBERSON

Department of Mathematics, Humboldt State University, Arcata, California

MARY ANN MADEJ

Redwood National Park, Arcata, California

We develop a stochastic model for the transport of stored sediment down a river channel. The model is based on probabilities of transition of particles among four different sediment storage reservoirs, called active (often mobilized), semiactive, inactive, and stable (hardly ever mobilized). The probabilities are derived from computed sediment residence times. Two aspects of sediment storage are investigated: flushing times of sediment out of a storage reservoir and changes in the quantity of sediment stored in different reservoirs due to seasonal sediment transport into, and out of, a reach. We apply the model to Redwood Creek, a gravel bed river in northern California. Although the Redwood Creek data set is incomplete, the application serves as an example of the sorts of analyses that can be done with the method. The application also provides insights into the sediment storage process. Sediment flushing times are highly dependent on the degree of interaction of the stable reservoir with the more mobile sediment reservoirs. The most infrequent and highest intensity storm events, which mobilize the stable reservoir, are responsible for the long-term shifts in sediment storage. Turnover times of channel sediment in all but the stable reservoir are on the order of 750 years, suggesting this is all the time needed for thorough interchange between these sediment compartments and cycling of most sediment particles from the initial reservoir to the ocean. Finally, the Markov model has adequately characterized sediment storage changes in Redwood Creek for 1947–1982, especially for the active reservoir. The model replicates field observation of the passage of a slug of sediment through the active reservoir of the middle reach of Redwood Creek in the 18 years following a major storm in 1964 that introduced large quantities of landslide debris to the channel.

INTRODUCTION

Recently, several investigators have pointed to the disparity between drainage basin erosion rates, measured on the upland slopes, and drainage basin sediment yield, measured at a gage at a downstream cross section [*Trimble*, 1977, 1981; *Meade*, 1982]. The disparity is because part of the sediment entrained in the upper portions of a basin frequently goes into storage further downslope or downstream. These storage sites are initially both a sink for entrained sediment and, at a later date, a major source for sediment moved further down a basin. Both *Trimble* [1981] and *Meade* [1982] have pointed out that sediment in storage in the channel can be the major source of erodible material in basins that had previously undergone severe upland erosion and consequent deposition in midbasin storage sites.

Other investigators, most notably *Gilbert* [1917], documented that rapid increases in stored channel sediment, brought on by sudden influxes of debris from hillslopes (in Gilbert's case due to hydraulic mining), can result in the downstream movement of such sediment as a wave or slug. The migrating sediment wave is confined to the active channel and dissipates in height and extends in length with movement downstream.

Paper number 6W4589.
0043-1397/87/006W-4589$05.00

Gilbert [1971] as well as *Meade* [1982], also noted that sediment stored on flood plains (by overbank deposition or isolated by downcutting) tends to stay in storage much longer than the more accessible sediment in the active channel. Therefore flood plain sediment tended to be stable, eroding only by lateral corrasion of the river, whereas sediment stored in the active channel, if delivered rapidly to a channel reach, is subject to further downstream movement as a wave or slug.

Meade [1982] claims that trying to predict the movement of stored sediment verges on a hopeless task because one obviously cannot assume a steady state for channel sediment reservoirs and, at the same time, the predictive period is too long to apply physical theories of sediment transport with any measure of success. However, the dynamics of stored sediment need not be treated as an analytic problem, nor as a deterministic problem only tractable with repeated field measurements. We present an alternative approach to the prediction of the movement of stored sediment and show that this alternative approach can account for both the unsteady movement of sediment particles and the movement of sediment particles as a wave that dissipates with time.

PURPOSE

This paper presents a method for developing a stochastic model for the movement of stored sediment down a river channel. In the first part we briefly outline the data requirements for the method and the general modeling approach,

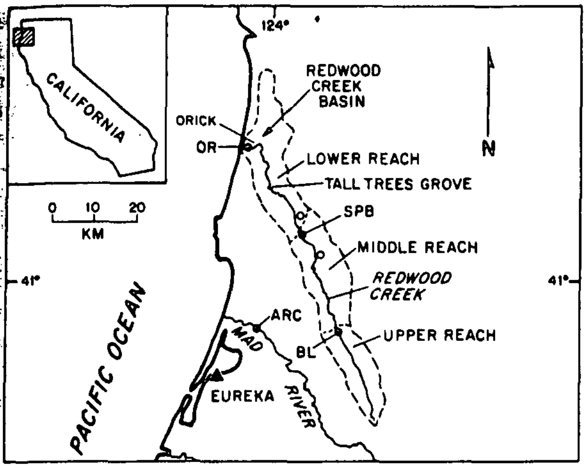

Fig. 1. Location map of the Redwood Creek basin showing the upper, middle, and lower study reaches and the location of the six U.S. Geological Survey gaging stations. Solid circles are main stem gaging stations (OR, Orick; SPB, South Park Boundary; BL, Blue Lake; ARC, Mad River station near Arcata), and open circles are tributary gaging stations.

which utilizes a finite Markov chain. In the second part of the paper we apply the method to Redwood Creek, a gravel bed river in northwestern California (Figure 1). This application of the method provides an opportunity to more clearly define the data requirements and to further develop the mathematical basis for four individual applications of the model. Although the Redwood Creek data set is incomplete and some approximations are necessary, the application serves as an example of the sorts of analyses that can be done using the method.

DATA REQUIREMENTS FOR THE MODEL

A brief description of data requirements follows. The second part of the paper includes a more thorough discussion of both data collection and interpretation as it applies to four specific applications.

Sediment Storage Reservoirs

A semiquantitative scheme to classify stored sediment in the field must be developed. The classification should be based on relative sediment mobility. Examples of the type of field data used to develop the classification include measurements of depth of scour during high discharges, age of vegetation growing on the deposit, and historic information on sediment mobility from aerial photographs. For the purpose of subsequent discussion, we will develop the method utilizing four sediment reservoirs of relative mobility, herein called active (frequently mobilized), semiactive, inactive, and stable (hardly ever mobilized). The volume of sediment in each of the defined sediment storage reservoirs must be known.

Bed Load Transport Data

Bed load transport data for gaged channel cross-sections are required. The distribution and number of gaging stations required depends on the size of the drainage basin and the channel reach that is being studied. Sufficient numbers of stations should exist to satisfy the following two conditions: (1) a relationship must be established between distance down the channel from the hydraulically most distant point and transport rate of particles in a specified reservoir, expressed in dimensions of M/T, and (2) for any channel reach of interest, the annual quantity of bedload entering the reach must be known for the period of interest.

Sediment Residence Time

Residence time for a reservoir is the mass M of sediment in a reservoir divided by the average transport rate Q_B of particles in the reservoir in dimensions of M/T. For the active reservoir, this average transport rate is usually derived from measurements taken in the field during high flows using a bed load sampler. The average rate that a particle moves while in the active reservoir is notably different from the average transport rate of all particles that make up the active reservoir. The latter rate is slower because of the probability that active particles may be intermittently stored in semiactive, inactive, or stable reservoirs. This slower rate is called the reservoir flushing time, which will be discussed further below.

Residence times are calculated for each sediment reservoir by a technique slightly modified from that of *Dietrich and Dunne* [1978]. Because the average transport rate within a reservoir is usually only known for the active reservoir, true residence times may only be determined for this reservoir. This problem is discussed in detail below in the application of the method.

DEVELOPMENT OF THE MODEL

Introduction

We attempt to model the dynamics of sediment storage through the use of a finite Markov chain. A finite Markov chain is a system in which there are a finite number of states and where movement from one state to another can be described only in terms of its probability. The model is based on the probability of transition of a particle of sediment between sediment storage reservoirs. These probabilities are derived from calculated sediment residence times.

The model addresses changes in volume (or mass) of stored sediment from one reach to the next. We chose this approach because it is exceedingly difficult to assess (even with extensive field measurements) the net total sediment input into, and discharge out of, a reach during a defined time period, or to estimate the time necessary for removing set volumes of bed load from a reach. The problem is complex because bed load size sediment in the channel and on adjacent depositional surfaces is not all equally accessible to transport. Some of the sediment is readily available for transport in the active channel, and, at the other extreme, some of the sediment resides in vegetated fill terraces from which it is transported only by infrequent, large floods. Furthermore, there is constant interchange during flooding events between the active reservoir and the less active reservoirs (Figure 2), so sediment may be highly mobile for a number of years, then through transport and redeposition in a major flood event, sediment may become virtually trapped for hundreds of thousands of years in relatively inaccessible flood deposits. The stochastic nature of the storms and floods, and the resultant complexity of movement of sediment between reservoirs, argues for a stochastic approach to the problem of modeling changes in sediment storage over time.

The state diagram in Figure 2 shows the basis for the Markov chain model as applied to sediment transport. For any one reach of stream, the arrows on the state diagram indicate the possible transitions between reservoir states.

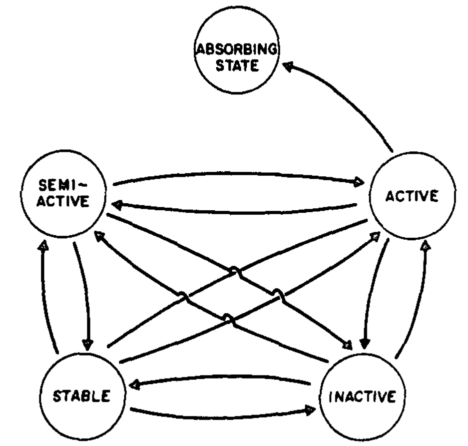

Fig. 2. State diagram showing the possible interchanges between the four sediment reservoirs. Note that sediment can only exit from the system (be absorbed) through the active reservoir. For Redwood Creek, the absorbing state is either a downstream reach, or the ocean, depending on whether the model addresses a single reach of the entire watershed.

Within any of the three study reaches, sediment in any one of the four reservoirs is free to interchange with any of the other reservoirs (Figure 2). Sediment may also exit from the system (a process called absorption), but (in this case) only through the active reservoir. Sediment may be absorbed in a downstream reach or in the ocean, depending on whether the model addresses a stream reach or the entire watershed. The major condition of movement is that the downstream motion of a particle of sediment depends only on the reservoir it is presently in, and particle motion does not depend on previous movement history, nor on the amount of time the particle has already been in its present reservoir state.

Mathematical Treatment

Our Markov chain model models the transport of stored sediment in discrete time steps of 1 year duration. For any given time interval, such as 1 year, we assume there is a fixed, known probability p_{ij} that a particle residing in state i at the beginning of the year will reside in state j at the beginning of the next year. These probabilities comprise the entries in the transitional matrix T. For example, if state 1 is the active reservoir in the upper reach of Redwood Creek and state 3 is the inactive reservoir for that reach, then p_{13} represents the fraction of sediment residing in the active reservoir at the beginning of a given year that one might expect to reside in the same reach but in the inactive reservoir at the end of that year. If $P_{ii} = 1$, then i is said to be an absorbing state and once a particle enters that state it can never leave it. All other states are known as transient states.

The states are numbered beginning with the absorbing states. As a result, the matrix has the general form

$$T = \left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right] \quad (1)$$

where I is an identity matrix (representing the absorbing state); 0 is a zero matrix; submatrix Q represents the probability movement among the various transient states; and R gives the probabilities for movement directly from a transient state to an absorbing state.

For the Redwood Creek study, there are 12 transient states representing four different sediment reservoirs in each of three reaches. In addition, there is one absorbing state, the ocean. The numbering of the transient states is organized so that the submatrix Q may be further broken down into submatrices Q_1, Q_2, and Q_3:

$$Q = \begin{bmatrix} Q_1 & \cdot & \cdot \\ 0 & Q_2 & \cdot \\ 0 & 0 & Q_3 \end{bmatrix} \quad (2)$$

which individually represent the transient states for the upper, middle, and lower reaches of a channel.

If we wish to study a single reach of the stream rather than the entire channel length, then the T matrix has the general matrix form (1) and

$$T = \left[\begin{array}{c|cccc} 1 & 0 & 0 & 0 & 0 \\ \hline r_1 & & & & \\ r_2 & & Q_i & & \\ r_3 & & & & \\ r_4 & & & & \end{array}\right] \quad (3)$$

where Q_i is the 4 × 4 submatrix of transient states just for that reach. The r_i's are chosen so the row sums will each be one. If sediment can only enter the absorbing state through the active reservoir (state 1), then r_1 will be the probability of that transition and r_2, r_3, and r_4 will be zero. For each of the three study reaches, we developed a transition matrix (Q_1, Q_2, Q_3) for sediment movement through the reach. The transition probabilities of the Q matrices are for movement in any 1 year period and are determined as follows. If we assume for reservoir i a residence time of n_i years, the probability that sediment will remain in that reservoir state after 1 year is $(n_i - 1)/n_i$ and the probability that sediment will leave the reservoir during the year is $1/n_i$. The transition probability p_{ii} must equal $(n_i - 1)/n_i$, and the sum of the remaining probabilities p_{ij} must equal $1/n_i$. For the transition matrices for each reach, the probabilities on the matrix diagonal are therefore computed directly from residence times, and the other probabilities of movement between the different states are partitioned from the relatively small probability of transition from one state to another state or to the absorbing state. Determination of these off-diagonal probabilities is discussed below. Aspects of our model are similar to a model of *Dacey and Lerman* [1983], who investigate sediment growth and aging as a Markov chain. The use of reservoir theory and transition probabilities for sediment reservoirs is conceptually discussed by *Dietrich et al.* [1982].

The volumes of sediment stored in the various states at the beginning of the nth year are recorded in the vector $D^{(n)}$. For example, in one of the examples (case 4) below, $D^{(1)}$ is the 1 × 4 row vector defining volumes of sediments as of 1947 in the active, semiactive, inactive, and stable reservoirs of the middle reach of Redwood Creek. Markov models have the property that

$$D^{(n)} = D^{(n-1)}T \quad (4)$$

That is, the distribution in year n is equal to the distribution in the previous year times the transition matrix. This assumes that no new sediment enters the system during that year. However, sediment does enter a channel reach each year, so we modify the model and let

$$D^{(n)} = D^{(n-1)}T + L^{(n-1)} \quad (5)$$

where the ith entry in the vector $L^{(n-1)}$ is the volume of new sediment residing in state i at the end of the year $n-1$ that was not accounted for in any of the volumes $d_i^{(n-1)}$. The case where $L^{(n)}$ is always the same has previously been discussed by *Lamberson* [1977].

Using the above procedure we generate four applications of the model (cases 1–4) that explore different aspects of the transport of stored sediment in Redwood Creek. An essential component of these applications is the computation of flushing times for sediment. Flushing time is the expected length of time for a particle starting in a certain reservoir to be flushed out to the absorbing state. Flushing time is significantly different from residence time, which is the average time a particle would take to leave a particular reservoir if it did not interact with other reservoirs. Because of reservoir interactions, actual flushing time is longer than residence time.

The fundamental matrix $N_t = (I - Q_t)^{-1}$ of Markov chains has the basic property that summation of the jth row in N_t gives the expected length of time for an event or particle starting in the jth state to be taken up by an absorbing state (matrix I of equation (1)). For a discussion of the fundamental matrix, see *Hayman and Sobel* [1982, pp. 256–259]. In the case of sediment particles in a river channel, the fundamental matrix allows the computation of particle flushing time for a given Q matrix. The sum of the jth row of $(I - Q_t)^{-1}$ is the expected number of years for a sediment particle starting in the jth reservoir (for example, the semiactive reservoir) to be flushed to the ocean or the next downstream reach. By definition therefore flushing time is the row sum of the jth row of the fundamental matrix $(I - Q_t)^{-1}$.

Our model will deal with two aspects of sediment stored in the main channel: flushing times of sediment out of a storage reservoir (cases 1–3) and the changes in the quantity of sediment stored in different reservoirs due to variable seasonal bedload transport into, and out of, a channel reach (case 4). The latter aspect addresses the migration downstream of a wave of sediment as described by *Gilbert* [1917] and as further discussed by *Mosley* [1978], *Kelsey* [1982], *Madej* [1982], and *Meade* [1982].

APPLICATION OF THE MODEL

The advantages and limitations of the method are best demonstrated by applying the method to a drainage basin that satisfies the data requirements outlined above. We apply this approach to Redwood Creek, a gravel bed river in north coastal California (Figure 1), because it has an extensive data base. Sediment storage changes and sediment transport are sufficiently well defined for the period 1947–1982 so that our stochastic modeling of stored sediment changes could be tested by independent data.

The application serves three purposes: (1) it demonstrates the analyses that can be done with the method, (2) it points to the steps in the method where data approximations are often necessary because particular data are difficult to obtain, and (3) it provides insight into the process of the movement of stored sediment down a river channel.

Redwood Creek: Study Watershed for the Model

The Redwood Creek basin (720 km^2) is an elongate, north-northwest-trending basin with a single 108 km long main stem channel that has an average gradient 1.5%. The channel is gravel bedded with a median grain size (d_{50}) ranging from 90 mm in the upper reaches to 16 mm near the mouth. Sediment transport rates reflect the rapid rates of erosion in this area: 2700 mg km^{-2} yr^{-1} in the upper basin (gaging station BL, Figure 1) and 2200 mg km^{-2} yr^{-1} at the mouth (gaging station OR, Figure 1). Mean annual precipitation is 2000 mm, most of which falls between October and March.

The period of adequate aerial photographic coverage of the Redwood Creek basin spans from 1947 to the present. Using the photographic data in conjunction with field measurement, various authors have documented the changing character of the main channel and the immediate streamside hillslopes since 1947 [*Harden et al.*, 1978; *Nolan and Janda*, 1979; *Kelsey et al.*, 1987; S. M. Colman, unpublished manuscript, 1973].

Major storms in December 1955 and December 1964 [*Waananen et al.*, 1971; *Harden et al.*, 1978] initiated numerous streamside debris slides along Redwood Creek. The 1964 storm generated the vast majority of streamside landsliding, delivering 2×10^6 m^3 of landslide debris to the upper reach alone [*Kelsey et al.*, 1987]. Comparing the dates and magnitudes of prehistoric floods in northern California [*Helley and LaMarche*, 1973; *Kelsey*, 1980; *Zinke*, 1981] with the 1964 flood suggests that a 1964 flood magnitude has a recurrence interval of 60–80 years. Net deposition in the main channel as a result of the 1964 storm was approximately 4.7×10^6 m^3. The increased volume of main channel sediment in Redwood Creek was initially concentrated in the upper reach, where most landsliding occurred. During the 1964 flood, gravel berms up to 9 m high were deposited in this reach. During the succeeding two decades after 1964, the sediment gradually migrated downstream. Major transport and deposition events occurred during storms in 1972 and 1975. The changes in volume and loci of sediment storage in Redwood Creek are well documented [*Nolan and Janda*, 1979; *Madej*, 1984; *Varnum*, 1984], and this documentation provides an opportunity to test the model.

For this study, Redwood Creek (excluding the lowest 3.6 km) was split into an upper, middle, and lower reach (35.5, 33.3, and 35.6 km in length, respectively). U.S. Geological Survey gaging stations are located at the downstream end of each of these reaches (Figure 1). Two additional gaging stations are located on tributary basins (Figure 1).

Sediment Reservoirs in Redwood Creek

During the 1979 and 1980 field seasons, all sediment stored in the main channel of Redwood Creek was mapped and classified into one of four reservoirs: active, semiactive, inactive, or stable (Figure 3). The stored sediment was classified on the basis of (1) relative position and elevation in comparison to the active channel, (2) density and age of vegetation growing on the deposit, and (3) presence of partially buried trees or artifacts such as logging cable or cut wood. The estimated discharge recurrence intervals used to classify stored sediment are based on vegetation age classes. Scour depths for active sediment are determined from both scour chain and high stage discharge measurements [*Madej*, 1984].

Active sediment, in the presently active main channel, moves with flows of a recurrence interval (RI) of 1–5 years. This sediment is unvegetated or very sparsely vegetated. The active reservoir includes bed material down to the annual depth of scour. Semiactive sediment is mobilized by flows with a RI of approximately 5–20 years. This sediment is directly

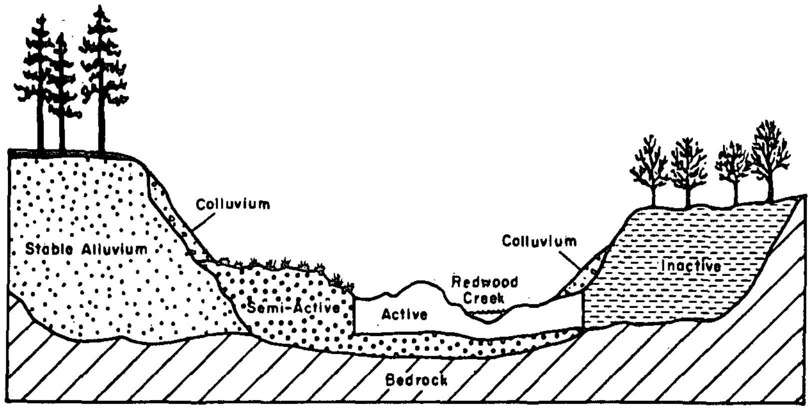

Fig. 3. Four sediment reservoirs in the Redwood Creek channel, based on the relative mobility of sediment. Positions of active, semiactive, inactive, and stable reservoirs are based on the relative position and elevation of stored sediment in comparison to the active channel.

adjacent to but higher than the active channel and is often covered with shrubs and young trees. Inactive sediment is mobilized by flows with a RI of approximately 20–100 years. These flows can mobilize coarse lag deposits, 3–5 m high gravel flood berms, intact log jams, and flood plain deposits. The gravel flood berms deposited during the 1964 storm are composed of inactive sediment.

Stable sediment consists of well vegetated flood plain deposits high above the thalweg. These deposits move extremely infrequently. In Redwood Creek, most stable sediment has not been mobilized historically except through localized lateral erosion. Some fine-grained material was added to the surface of stable reservoirs during the 1955 and 1964 floods. The thick early to mid-Holocene valley fill in lower Redwood Creek was not included in the stable sediment volumes.

Stable sediment may be influenced by tectonic activity due to the generally long intervals between its mobilization. Uplift rates in Redwood Creek are on the order of 1 m/10^3 yr [*Kelsey*, 1987], so there is a possibility that stable sediment could be isolated by uplift and become the alluvial veneer on a strath terrace. Although strath terraces are prevalent in several reaches along Redwood Creek, the total volume of sediment incorporated in these straths in only a few percent of the total stable reservoir volume. Therefore even though uplift does isolate stable sediment and effectively remove it from alluvial transport, the volume involved is insignificant.

In addition to the sediment reservoir volumes measured in the field in 1979–1980, volumes for 1947 and 1965 (after the December 1964 storm) were calculated using aerial photographs and previous channel cross-section surveys. Volumes were converted to mass using a mass density ρ of 1.92 g/cm^3.

Calculation of Sediment Residence Time for Redwood Creek

Calculations of residence time require knowledge about the average transport rate Q_B of particles in the reservoir in question. The average transport rate for the active reservoir of Redwood Creek was derived from periodic measurements of bed load discharge during 1974–1982 by the U.S. Geological Survey [*U.S. Geological Survey*, 1974–1982]. Bed load measurements used a Helley-Smith bed load sampler at moderate discharges. At higher discharges, bed load discharge was calculated using the Meyer-Peter-Muller and modified Einstein bed load formulas. Average bed load transport rate within a reservoir is only known for the active reservoir, and true residence times can only be determined for this reservoir. To compute residence times for the other reservoirs, increasingly greater reservoir masses are divided by the same average transport rate of particles in the active reservoir Q_B. This computational method is a surrogate for computing the three residence times of the successively less active reservoirs, using the particular reservoir mass and successively smaller (and unknown) average particle transport rates. For example, the residence time for semiactive sediment is $[M_{\text{active}} + M_{(\text{semiactive})}]/Q_B$, where M is reservoir mass in megagrams and Q_B is sediment discharge in megagrams per year. By this procedure, computed residence time for the semiactive, inactive, and stable reservoirs is an index of the size of the reservoir. The fact that Q_B is only known for the active reservoir and residence times of the other reservoirs must be calculated without knowing Q_B for those reservoirs is a weak point in the development of the stochastic model. Better methods to calculate Q_B, given more field data, will be discussed in the last section. This approximation technique for residence times was necessary to calculate transition probabilities. The technique appears to give credible residence times in light of the field evidence (further discussed below).

Calculations of residence times for the reservoirs in Redwood Creek follow the methods of *Dietrich and Dunne* [1978]. Using available data for the entire basin, sediment mass per unit channel length M_s and bed load discharge Q_B were defined as power functions of main stem channel length X (Table 1):

TABLE 1. Definition of Power Functions Used to Compute Sediment Residence Time

Reservoir	Relation	a	b	p	n	r^2
Entire channel	$Q_B = bX^n$		1738		0.97	0.86
Ac sediment	$M_s = aX^p$	1.89		0.86		0.57
Ac plus Sa sediment	$M_s = aX^p$	3.46		0.85		0.59
Ac plus Sa plus Ia sediment	$M_s = aX^p$	4.17		0.95		0.53
Ac plus Sa plus Ia plus St sediment	$M_s = aX^p$	4.60		1.57		0.53

Ac, active; Sa, semiactive; Ia, inactive; St, stable sediment reservoirs. Q_B, average transport rate (Mg/year); X, main stem channel length (km); M_s, mass per unit channel length (Mg).

TABLE 2. Sediment Residence Times for Sediment in Each of the Four Reservoirs in Each of the Three Reaches

Reservoir	Reach	Residence Time, years
Active	U	14
	M	11
	L	11
Semiactive	U	23
	M	18
	L	18
Inactive	U	70
	M	65
	L	68
Stable	U	1300
	M	1900
	L	2400

U, upper reach; M, middle reach, L, lower reach.

$$M_s = aX^p \tag{6}$$

$$Q_B = bX^n \tag{7}$$

where a, b, p, and n are constants. Four separate power functions for the four different reservoir masses (Table 1) were derived to define the relationship of the mass of sediment to channel length (equation (6)). The relationship in (6) is based on field data collected from 38 study reaches that ranged in length from 1.1 to 7.5 km. These reaches encompassed the total length of Redwood Creek with the exception of a bed rock gorge that stored virtually no sediment. The volume of sediment stored in each reservoir was measured in each of the 38 reaches. The relationship in (7) was based on bed load transport data at the three main stem and two tributary gaging stations on Redwood Creek and on similar data from a gaging station at the lower end of the Mad River (Figure 1). Computation of power function relationships is further discussed by *Madej* [1984].

Combining (6) and (7) yields the following relationship between residence time per unit channel length ($dt/dx = M_s/Q_B$) and the power functions of (6) and (7):

$$\frac{dt}{dx} = \frac{M_s}{Q_B} = \frac{a}{b} X^{p-n} \tag{8}$$

Integration of (8) allows calculation of residence time for a reach of channel of length $x_2 - x_1$:

$$\int_{t_1}^{t_2} dt = \int_{x_1}^{x_2} \frac{a}{b} X^{p-n}\, dx \tag{9}$$

(modified from *Dietrich and Dunn* [1978]). In such a manner, using the values for the constants in Table 1, residence times for the four reservoirs were computed for three reaches of Redwood Creek (Table 2). Sediment in the active reservoir has the shortest residence time. Residence times for the inactive reservoir are approximately 5–6 times greater than for the active reservoir, and residence time for the stable reservoir is approximately an order of magnitude greater than for the semiactive reservoir (Table 2).

Calculated values of residence time (Table 2) are generally consistent with the type of vegetation growing on the different reservoirs. Stable sediment supports coniferous forests, implying stability on orders of hundreds of years. Inactive sediment was mobilized by the 1964 storm (approximately a 60- to 80-year recurrence interval event), but these deposits have been otherwise stable and now support a young hardwood riparian forest. Semiactive sediment has moved on at least four occasions in the two decades since 1964. Therfore despite the assumptions we must make to calculate residence time for the three less active reservoirs, the calculated results are consistent with field observations and will be used in the next step of model development.

Residence time gives the average amount of time a particle would spend in a reservoir, based on the mobility of that reservoir. However, a residence time does not indicate how long any one particle will take to totally leave a reach. If a particle in the active reservoir is mobilized, it may be carried out of the reach (be absorbed), or it may be redeposited within the reach in another reservoir such as the inactive reservoir where it will reside for a much longer time before being remobilized and ultimately flushed out of a system. Therefore a residence time only provides the basis for assigning a probability that a particle of sediment will leave a certain state within a set time period. This probability, in turn, is the basis for computing sediment flushing time.

Four Applications of the Method to Redwood Creek

Sediment flushing times out of each of the three mainstem reaches (case 1). In our first analysis (case 1) we compute

4a:

Upper reach:

$$Q_1 = \begin{array}{c} \\ \end{array}\begin{bmatrix} .9286 & x & x & x \\ x & .9565 & x & x \\ x & x & .9857 & x \\ x & x & x & .99923 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array} \quad (\text{columns: } A,\ Sa,\ I,\ St)$$

Middle reach:

$$Q_2 = \begin{bmatrix} .9091 & x & x & x \\ x & .9444 & x & x \\ x & x & .9846 & x \\ x & x & x & .99947 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array}$$

Lower reach:

$$Q_3 = \begin{bmatrix} .9091 & x & x & x \\ x & .9444 & x & x \\ x & x & .9853 & x \\ x & x & x & .99958 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array}$$

4b:

Upper reach:

$$Q_1 = \begin{bmatrix} .9286 & .0150 & .0050 & .0014 \\ .0239 & .9565 & .0174 & .0022 \\ .0064 & .0064 & .9857 & .0015 \\ .00016 & .00035 & .00026 & .99923 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array} \quad (\text{columns: } A,\ Sa,\ I,\ St)$$

Middle reach:

$$Q_2 = \begin{bmatrix} .9091 & .0191 & .0064 & .0018 \\ .0306 & .9444 & .0222 & .0028 \\ .0070 & .0069 & .9846 & .0015 \\ .00011 & .00023 & .00019 & .99947 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array}$$

Lower reach:

$$Q_3 = \begin{bmatrix} .9091 & .0191 & .0064 & .0017 \\ .0306 & .9444 & .0222 & .0028 \\ .0067 & .0065 & .9853 & .0015 \\ .00008 & .00019 & .00015 & .99958 \end{bmatrix} \begin{array}{l} A \\ Sa \\ I \\ St \end{array}$$

Fig. 4. Transition matrices (Q_1, Q_2, Q_3) for the upper, middle, and lower reaches respectively. (*a*) Matrices Q_1, Q_2, and Q_3 showing just the diagonal probabilities that are computed directly from the residence times. Other off-diagonal probabilities, denoted by x, are more difficult to evaluate and are the subject of case 1 application of the method. (*b*) Matrices Q_1, Q_2, and Q_3 determined from residence times for the sediment reservoirs plus trial runs of study reach flushing times. A, active reservoir; Sa, semiactive reservoir; I, inactive reservoir; St, stable reservoir. Note that for each matrix the row sum of Sa, I, and St equals 1, whereas the row sum of A is less than 1, the difference being equal to the probability of a sediment particle being absorbed out of the active reservoir.

TABLE 3. Average Number of Years to Empty Initial Reservoir (Sediment Flushing Times From the Reservoir) Given Varying Amount of Interaction With the Stable Reservoir

Initial Reservoir of Sediment Particle	Lower Reach Flushing Times			Middle Reach Flushing Times			Upper Reach Flushing Times		
	1	2	3	1	2	3	1	2	3
Active	246	616	917	204	502	741	166	369	538
Semiactive	602	1620	2370	492	1320	1910	399	967	1380
Inactive	749	1820	2690	608	1500	2170	499	1100	1570
Stable	2970	3750	4340	2360	3030	3470	1680	2150	2450

Here 1 is minimal interaction, 2 is moderation interaction, and 3 is substantial interaction.

flushing times for sediment out of each of the three study reaches using $Q_i = Q_1$, Q_2, and Q_3. To compute flushing times for case 1 and subsequent cases we first had to determine the off-diagonal matrix probabilities (Figure 4*a*), representing the probabilities of movement between reservoir states. The row sum of these probabilities is in all cases small (always smaller than 0.0909 and averages 0.0378 for the 12 possible cases), but the determination of accurate off-diagonal matrix probabilities is critical because these probabilities determine reservoir flushing times. It is not possible to compute these probabilities from available field measurements. However, comparison of cumulative distance of actively eroding channel banks among the four reservoirs, as well as data on how frequently the reservoirs are mobilized (see above), provide a basis for estimating a range of possible exchange probabilities among reservoirs.

We formulated three different sets of Q matrices (three different sets of off-diagonal probabilities) that represent different degrees of interaction (minimal, moderate, and substantial) with the stable reservoir. Because the stable reservoir in the short term is a sink (due to extremely long residence times, Table 2), it can substantially influence flushing times. The resulting flushing times (Table 3) show that flushing time is sensitive to partitioning of the off-diagonal matrix probabilities. As an example of the analytical method employed in calculating the values of Table 3, we show in the appendix the Q and N matrices used in case 1 for calculating flushing times for moderate interaction with the stable reservoir.

Insufficient field data prevent us from unequivocally choosing the best Q matrix from the three used to compute the flushing times in Table 3. Lacking this data, we determined which result appears to be most reasonable. We evaluated residence time compared to flushing time estimates for the stable reservoir (Table 2 versus Table 3). Based on the definition of flushing versus residence times, the two should be the most similar for the stable reservoir because stable reservoir particles only interact with more active reservoirs and the probability of such interaction is quite small. Using this criteria, the flushing times based on minimal interaction with the stable reservoir are the most similar to stable reservoir residence times. Based on these results, we selected Q_1, Q_2, and Q_3 with minimal interaction with the stable reservoir to be the transition matrices (Figure 4*b*) that are used in all succeeding models (with slight modification for cases 2 and 4).

Minimal interaction with the stable reservoir cannot be precisely defined in terms of frequency of mobility. However, minimal interaction implies that the stable reservoir only moves during climatically extreme events because it was only mobilized to a minor degree by isolated incidences of bank erosion during the 1964 flood.

In a physical sense, through varying the extent of activity of the stable reservoir, we modeled different ways that stable sediment can move down a specified reach length. Storm events are the cause of major episodes of stable sediment transport. The flushing time results (Table 3) therefore compare the effects of a few major storms that move stable sediment great distances but occur infrequently (minimal interaction with stable reservoir) and a number of frequent, smaller storm events that move stable sediment often (substantial interaction with stable reservoir) but never move it very far. The results suggest that infrequent, high-intensity flood events, which mobilize the stable reservoir, are the type of events that are most responsible for the major long term shifts in sediment storage in Redwood Creek. In light of this, it is noteworthy that even though the 1964 flood brought major geomorphic change to this basin, it essentially did not mobilize the stable reservoir.

Sediment flushing times to ocean from twelve different initial sediment reservoirs along Redwood Creek (case 2). The second analysis (case 2) determines the expected length of time for a particle starting in any of the 12 initial reservoirs (four reservoirs in each of the three reaches) to be flushed to the ocean. The second analysis therefore uses a 12 × 12 Q matrix, which is in the general matrix form (2). The flusing times for sediment moving from any state in the system to the ocean in this model are given by the appropriate row sum in N, where $N = (I - Q)^{-1}$.

Once the expected flushing times are computed we can compute their variance by evaluating the matrix equation:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = N \begin{bmatrix} 2f_1 - 1 \\ 2f_2 - 1 \\ \vdots \\ 2f_n - 1 \end{bmatrix} \quad (10)$$

where v_i is the variance given that the sediment began in state 1, the f_i's are the flushing times, and N is the appropriate matrix [*Isaacsen and Madsen*, 1976]. The variance here is the variance in time of transit for individual particles flushed through the system, as predicted by the model. The variance is due to different visit times of particles in the four different reservoirs.

The Q transition matrix (Figure 5) is derived from Q_1, Q_2, and Q_3 (Figure 4) but is slightly modified from Q_1, Q_2, and Q_3 in order to reflect an increased number of transition probabilities among reservoirs. The greater number of possible transition in this composite model allows sediment to enter a downstream reach from an upstream reach through the semiactive and inactive reservoirs as well as through the active reservoir. In addition, sediment can enter the ocean (be absorbed) from the lower reach through both the active and semiactive reservoirs. Therefore for 6 of the 12 initial states the modified model allows sediment to travel to the next downstream reach in one year. The off-diagonal probabilities in these cases were altered from Figure 4*b* and were designated according to the priority scheme in Figure 5. The priority scheme ranks transitions from the most likely to the least likely based on qualita-

	(A_1)	(Sa_1)	(Ia_1)	(St_1)	(A_2)	(Sa_2)	(Ia_2)	(St_2)	(A_3)	(Sa_3)	(Ia_3)	(St_3)
(A_1)	0.9286	0.01506	0.00502	0.00144	0.04035	0.00744	0.00186	0.00026	0	0	0	0
(Sa_1)	0.01643	0.9565	0.01656	0.00034	0.0068	0.00127	0.00207	0.00004	0	0	0	0
(Ia_1)	0.00614	0.00509	0.9857	0.00006	0.00141	0.00129	0.00024	0.00007	0	0	0	0
(St_1)	0.0002	0.00054	0.00003	0.99923	0	0	0	0	0	0	0	0
(A_2)	0	0	0	0	0.9091	0.0191	0.00636	0.0018	0.0512	0.00954	0.00254	0.00036
(Sa_2)	0	0	0	0	0.02168	0.9444	0.02113	0.00039	0.00890	0.00252	0.00093	0.00005
Q = (Ia_2)	0	0	0	0	0.00622	0.00370	0.9846	0.00017	0.00310	0.00149	0.00057	0.00015
(St_2)	0	0	0	0	0.00011	0.00004	0.00038	0.99947	0	0	0	0
(A_3)	0	0	0	0	0	0	0	0	0.9091	0.02	0.0159	0.002
(Sa_3)	0	0	0	0	0	0	0	0	0.02	0.9444	0.02	0.0049
(Ia_3)	0	0	0	0	0	0	0	0	0.00670	0.00623	0.9853	0.00178
(St_3)	0	0	0	0	0	0	0	0	0.00008	0.00019	0.00015	0.99958

initial state	possible transitions in descending order of priority						
A_1	A_2	Sa_1	Sa_2	Ia_1	Ia_2	St_1	St_2
A_2	A_3	Sa_2	Sa_3	Ia_2	Ia_3	St_2	St_3
Sa_1	A_1	Ia_1	A_2	Sa_2	Ia_2	St_1	St_2
Sa_2	A_2	Ia_2	A_3	Sa_3	Ia_3	St_2	St_3
Ia_1	A_1	Sa_1	A_2	Sa_2	Ia_2	St_1	St_2
Ia_2	A_2	Sa_2	A_3	Sa_3	Ia_3	St_2	St_3

Fig. 5. (Top) A 12 × 12 transition matrix (Q matrix) used for the flushing time analysis of case 2. The twelve transient states constitute the four different reservoirs in each of the three study reaches. (Bottom) Tabular presentation of priority scheme for assigning transition probabilities for different initial reservoir states. A, active; Sa, semiactive; I, inactive; St, stable. Varying number of significant figures reflects the fact that all row summations must equal one except for A_3 and Sa_3.

tive field observations of reservoir interaction and on reservoir residence times. It was not possible to quantitatively determine the off-diagonal probabilities for the 12 × 12 Q transition matrix from field data. Because these probabilities are not more than professional judgments based on field observations, the calculated flushing times (Table 4) are necessarily suspect. We did a sensitivity test to determine if the flushing times vary significantly with variation in the off-diagonal probabilities. The first part of the sensitivity test was conducted by varing the off-diagonal probabilities as much as possible, subject to only two constraints: (1) the diagonal probabilities (computed directly from the residence times) were held constant and (2) the hierarchy of possible transitions from an initial state to a final state after 1 year (bottom, Figure 5) were not changed. This test, consisting of seven different model runs with different off-diagonal probabilities, revealed that the flushing times did not differ within the rounding error of 50 years. Two of the model runs in the above sensitivity test allowed sediment from the stable reservoir, as well as all other reservoirs, to move to the next lowest reach in 1 year. The second part of the sensitivity test was conducted by varying the diagonal probabilities to reflect larger residence times for all the sediment reservoirs. The off-diagonal probabilities were again assigned according to the hierarchy of possible transitions in Figure 5. Even with this significant change, flushing times for the three more active reservoirs were within 35% of the values in Table 4, and flushing times for the stable reservoir were approximately twice the values in Table 4.

TABLE 4. Expected Length of Time for Particle Starting in Specified Reservoir to be Flushed to the Ocean

Initial Reservoir	Flushing Time, years: Mean	Flushing Time, years: Standard Deviation
U-A	750	1950
U-Sa	800	1950
U-I	850	2000
U-St	2100	3050
M-A	650	1850
M-Sa	700	1900
M-I	750	1950
M-St	2600	3700
L-A	450	1550
L-Sa	800	2150
L-I	1000	2350
L-St	3200	4450

Flushing time computed value was rounded off to nearest 50 years. U, upper reach; A, active reservoir; Sa, semiactive reservior; I, inactive reservoir; St, stable reservoir; M, middle reach; L, lower reach.

Results of the sensitivity test indicate that knowing precise values for all the off-diagonal transition probabilities is not critical, if the residence time estimates are accurate. This conclusion is important because it would be highly unusual for field data to be extensive enough to directly compute the off-diagonal probabilities of Figure 5. Even if residence times are slightly in error, flushing times are also not particularly sensitive to slight modifications in the diagonal probabilities. The sensitivity test takes on additional importance because the results of the model run for case 2, presented below, are not particularly intuitive.

Flushing times of Table 4 suggest that for the active, semiactive, and inactive reservoirs, the expected time a particle arrives at the ocean is not significantly different, regardless of which reach the particle started in and irrespective of whether the particle started in the active, semiactive, or inactive reservoir. Therefore for the Redwood Creek channel, the average length of time for particle turnover in any one of these reservoirs is approximately 750 years. Variability for the individual flushing times is quite large, indicated by the standard deviations about the mean (Table 4). This large variability is expected given the relatively long residence time for particles in the middle and lower reach stable reservoirs (Table 2).

The stable reservoirs can retain sediment for times on the order of 2000–4000 years (Tables 3 and 4). Stable sediment reservoirs in the upper two thirds of Redwood Creek take the form of heavily forested terraces along the channel. Near the mouth, forested flood plain deposits on top of the Holocene valley fill constitute the stable reservoir. Two ^{14}C dates on wood from the stable reservoir at the base of an alluvial fill at the Tall Trees Grove (Figure 1) provide independent evidence that stable reservoirs retain sediment up to 4000 years. The two samples of wood yield ages of 3580 ± 70 and 3520 ± 100 ^{14}C years before present (D. K. Hagans, Redwood National Park, unpublished data, 1987).

The results of case 2 (Table 4) further show that for active sediment, flushing times are longest if the sediment starts in the upper reach and shortest for active sediment starting in the lower reach. However, as the reservoir becomes less likely to be mobilized, the situation reverses. For the stable reservoir, the relatively small amount of sediment that starts in this reservoir in the upper reach is likely to reach the ocean before the large amount of stable sediment that is in more permanent storage in the lower reach. These results, however, are a direct consequence of the probabilities in the Q matrix of Figure 5.

Sediment distribution and depletion in the four middle reach reservoirs over time (*case 3*). It is worth looking further at the flushing time model for a single study reach, such as the middle one, and compare the flushing time to the transit time for individual sediment particles moving through a reservoir. Transit time is the time required for an individual particle to be flushed through the system, whereas flushing time is the mean of all transit times of all particles in the reservoir. For case 3 we continuously load the same amount of sediment into the active reservoir of the middle reach on a yearly basis. In (5), $L^{(n-1)} = (1, 0, 0, 0)$ and

$$T = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline r_1 & & & \\ r_2 & & Q_2 & \\ r_3 & & & \\ r_4 & & & \end{array}\right] \qquad (11)$$

Matrix Q_2 is that shown in Figure 4*b*. $D^{(n)}$ (see equation (4)) in this analysis gives the sediment distribution through the four reservoirs after n years assuming continuous loading.

The Markov model will yield the distribution of sediment among reservoirs and depletion of sediment from reservoirs in the reach at any given year after input starts (Table 5). For instance, 35 years (the time span of the case 4, discussed below) after we started to trace the sediment entering the middle reach, 33% was in the active reservoir, minor amounts were in the other reservoirs, and 50% had been absorbed into the lower reach. However, the average flushing time for sediment entering the active reservoir of the middle reach is 204 years (Table 3). The discrepancy between a 204-year average

TABLE 5. Distribution of Sediment With Time Among the Four Reservoirs and the Absorbing State Given the Case of Continuous Loading of Sediment into the Active Reservoir, Middle Reach of Redwood Creek

Years Since Sediment Input	Distribution of Sediment, % of Total				
	Active	Semi-active	In-active	Stable	Absorbed
5	84	3	1	0	12
10	68	6	2	1	23
15	57	7	4	1	31
20	48	8	5	1	38
25	42	8	5	2	43
30	37	8	6	2	47
35	33	8	7	2	50

flushing time and a transit time through the system of 35 years for 50% of this sediment is seemingly problematical. However, the first is the mean transit time while the second is the median transit time. The difference is explained by the fact that if a particle from the active reservoir happens to enter the stable reservoir, it can stay there thousands of years, substantially influencing the mean transit time. In the above case, random visits of active reservoir particles to less active reservoirs results in the mean transit time for such particles (flushing time) being almost 6 times the median transit time. This result has significance for sediment tracer studies where only a fraction of marked particles are recovered. A recovered marked particle most likely will travel near the median transit time but will not furnish information on the mean transit time (which should be much longer) for particles in that reservoir population.

A model of sediment storage changes in the middle reach for 1947–1982 (*case 4*). The final application (case 4) involves modeling the changes in Redwood Creek storage volumes of the middle reach reservoirs during the period 1947–1982. We use the same transition matrix for the middle reach (matrix Q_2, Figure 4*b*), except that during the study period, the stable reservoir was not eroded at all, and the inactive and semi-active probabilities are increased accordingly. Data generation is as follows. At the start of year 1947, there are four initial reservoir volumes. These volumes constitute a 1 × 4 row matrix, which is multiplied times the transition matrix to give a new set of reservoir volumes. The bed load input for the succeeding year is then added to the active reservoir giving reservoir volumes at the start of year 1948. These volumes are then multiplied times the transition matrix, and so on. This procedure is reiterated 36 times to give reservoir volumes at the start of each year from 1947 to 1982.

Case 4 predictes the changes in the amount of sediment storage in different reservoirs due to seasonal bed load transport into, and out of, a channel reach. As a test of the capability of such a model, we used sediment volume and bed load transport data for the middle reach of Redwood Creek for the 36-year period 1947–1982. The period also encompasses the two major storms in 1955 and 1964. For the model we know the amount of sediment in each of the four middle reach reservoirs as of 1947 based on aerial photographs and field mapping, and we calculated the annual addition of bed load sediment into the top of the middle reach for the years 1947–1982,

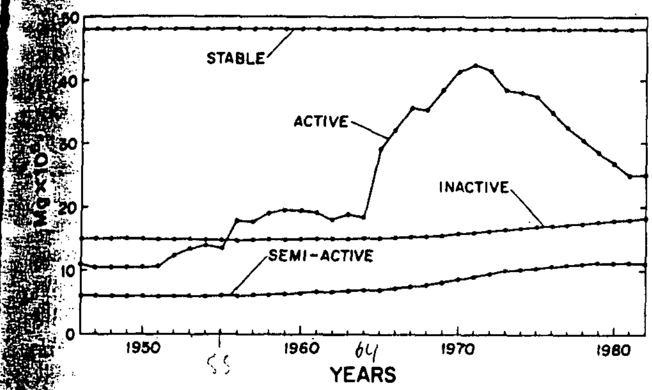

Fig. 6. Mass of sediment in reservoirs (in megagrams) in the middle reach of Redwood Creek for 1947–1982. Note that the stable reservoir was not mobilized during this period.

based on measurements at gaging station BL (Figure 1).

Calculations of the annual bed load increment used the following procedure. For years 1974–1982, bed load discharge was calculated directly from bed load transport curves for those years in conjunction with flow duration curves for the gaging station. For the years of bed load sampling, calculated annual bed load averaged 31% of total annual load. For 1947–1973, annual bed load discharge was calculated from suspended sediment discharge assuming bedload to be 31% of total load. Suspended sediment was sampled at the U.S. Geological Survey gaging near Blue Lake (BL, Figure 1) only during station operation from 1954–1958 and 1974–1982, so suspended sediment from 1947–1954 and 1959–1973 was estimated from suspended sediment records at a gage in an adjacent basin of similar size (Van Duzen River at Bridgeville, California) normalized to the suspended sediment discharge at the Redwood Creek gage (BL, Figure 1) in 1958 (the only year both gages were operating concurrently). In addition, an additional component of bed load was added for the 8 years following the 1964 flood, based on field volume measurements of the amount of coarse sediment that was transported out of the upper reach of Redwood Creek in the 8 years following the 1964 flood. This additional component reflects the shift in the suspended sediment discharge relationship after the flood such that a given discharge carried more suspended sediment than in preflood conditions. Although a lack of data precludes documentation of this shift in Redwood Creek, the shift occurred in all adjacent river basins that had sediment discharge measurements before and after the 1964 flood [*Knott*, 1971; *Brown*, 1975].

The model results show yearly incremental changes in mass of the three more active reservoirs in the middle reach for 1947–1982 (Figure 6). The stable reservoir has not moved historically and shows no change. Changes in the active reservoir initiated by the 1955 and 1964 storm events are apparent (Figure 6).

We only used the mass of the 1947 reservoir to generate these results, and the model yields 1965 and 1980 reservoir masses that are very close to the field measured values (Table 6). These results indicate a reasonable degree of accuracy for the model. However, it must be recognized that residence times were computed using the mass of the 1980 reservoirs, so the model was calibrated with 1980 data, then run with 1947 data, and the computed 1965 reservoir mass data is the only independent test of model accuracy. The model suggests that

TABLE 6. Modeled Total Amount of Sediment in Sediment Reservoirs Versus Field Measurement Total Amount of Sediment in Sediment Reservoirs

Method	1965 Total Sediment, Mg	1980 Total Sediment, Mg
Field measurement	10,597,000	11,280,000
Model estimate	9,924,000	10,400,000

the observed downstream-moving wave of sediment was mainly confined to the active reservoir. This observation is confirmed by the channel cross-section measurements of *Varnum* [1984].

Even though the field surveys and model results are similar, the field results show details the stochastic model does not. In comparison to modeled results (Figure 6), response of the semiactive and inactive reservoirs to the 1964 storm event was initially more abrupt. All three of the more active sediment reservoirs increased in mass immediately after the 1964 flood. The active and semiactive reservoirs decreased rapidly, but the mass of the inactive reservoir decreased more slowly. For example, in the upper reach, 30% of the increase in stored sediment due to the 1964 flood was due to increases in the inactive reservoir, and most of this sediment is still in storage as of 1986. In the middle reach, the effects of floods in the 1970s masked some of the evidence of the 1964 flood. Nevertheless, about 1.2×10^6 m^3 of 1964 flood-deposited sediment remains in the middle reach, primarily stored in semiactive and inactive reservoirs. Channel cross-sectional surveys from 1973 to 1986 show rapid flushing (3–10 years) of active and semiactive sediment from stream channel cross-sections [*Varnum*, 1984], but much slower removal times (> 10 years) where inactive reservoirs store much sediment. From 1973 to 1986, the removal of flood-deposited sediment in Redwood Creek ranged from 70–90% in narrow reaches with only active and semiactive reservoirs to only 0–13% in reaches where inactive reservoirs were the dominant storage features. Therefore the stochastic nature of the model led to gradual reservoir mass changes for the two less active reservoirs (Figure 5), whereas field surveys show that semiactive reservoir response varied with basin position and channel width and the inactive reservoir increased in mass rapidly but is decreasing to prestorm volumes gradually.

DISCUSSION

Our proposed method to model sediment storage changes in a river channel has limitations as well as providing insight into the dynamics of sediment storage. The data set needed to compute sediment residence times is not generally available for most drainage basins. Bed load transport rates must be estimated along several points in the channel, and initial volumes of stored channel sediment must be accurately estimated. The model will predict bed load movement in a reach only if a given bed load transport rate for each year is entered into the upper end of the study reach. The model then processes sediment through a reach based on probabilities of movement between reservoir states.

The method relies on the ability to calculate residence times of particles in four different sediment reservoirs (calculated residence time is the total mass of the reservoir M divided by the average transport rate of the reservoir Q_B in dimensions of

M/T). This is a considerable problem and we can only address it rigorously for the active reservoir, because we know Q_B for this reservoir only. For the other three reservoirs, we in essence assume the active reservoir is completely emptied before sediment grains are removed from the next most active reservoir (see above). This method produced acceptable results in that the residence times for nonactive reservoirs are reasonable in light of independent evidence of reservoir characteristics from vegetation ages and historic sediment transport events. However, reservoirs do not empty in such a fashion, but rather particles from less active reservoirs are supplied to the more active reservoirs by bank erosion and slope processes.

A more rigorous way to evaluate sediment residence times for the less active reservoirs would be to estimate the magnitude and frequency of bank erosion and mass movement within a reach. Such estimates, though crude, could be accomplished by repeated channel surveys, erosion pins, and interpretation of time-sequential aerial photography. Such studies are on-going in Redwood Creek. Other techniques that could provide better age estimates for channel stored sediment, leading to improved estimates of Q_B for the semiactive, inactive, and stable reservoirs, include further dating of reservoirs using ^{14}C techniques (although charcoal and wood are extremely scarce) and possibly cesium 137 [*Wilkin and Hebel*, 1982].

In addition to realistic residence times, the method relies on accurate off-diagonal transition probabilities between reservoir states. Because of a lack of detailed field data, these must be estimated. These estimates are narrowly constrained by the residence times and by relative sediment mobilization rates for the reservoirs. Nonetheless, computed flushing times for different reservoirs in any one reach are sensitive to the degree of interaction with the stable reservoir. It was therefore necessary to choose the best set of off-diagonal probabilities based on the most reasonable computed results. The lack of a better method of evaluating the off-diagonal probabilities for a reach is the other weak point in the application of the method and can be improved by more data on reservoir exchange rates. However, once off-diagonal probabilities for the three transition matrices for each of the three reaches are chosen, flushing times for sediment particle movement through all three reaches, using combined transition matrices, are not sensitive to changes in the off-diagonal probabilities. One aspect of our sensitivity analysis showed that even if the diagonal probabilities (i.e., the residence times) are changed, flushing times for particles through the three reaches remain in the same order of magnitude.

Another consideration in using the model is the origin of stored sediment in the basin of interest. Sediment in storage due to tectonic subsidence or due to different climatic regimes, such as glacial outwash, may have a residence time many orders of magnitude longer than the more active sediment moving through the present channel.

How well does the model simulate geomorphic process? Coarse sediment particles on the channel bed do move by a Markov process inasmuch as present movement is not dependent on previous movement history. To make the model workable, however, it is designed to trace the downstream movement of stored sediment on a reach-to-reach basis. In the model, sediment can only be delivered to the channel at the beginning of each study reach. Any midreach sediment contributions cannot actually be accounted for until the bed load input at the beginning of the next reach.

Given the limitations, the Markov model for tracing the movement of stored sediment particles provides insight into a number of aspects of sediment storage when the method is applied to Redwood Creek. These aspects include particle longevity in the channel, degree of interchange of particles among reservoirs given assigned transition probabilities, the type of events that move the stable reservoir, and movement of slugs of stored sediment. Considering the Redwood Creek main channel as a whole, the mean longevity of a particle (average flushing time) from any initial point in the channel until the time it leaves the system is on the order of 750 years for the active, semiactive, and inactive reservoirs. This somewhat surprising result was replicated using seven different sets of off-diagonal probabilities in the 12 × 12 transition matrix (Figure 5), suggesting the conclusions are not particularly sensitive to the exact values of these probabilities. Equal flushing times for particles starting in any one of these reservoirs implies that the average particle spends time in each of the three more active reservoirs in a 750-year period. The scatter about the mean flushing time value is large (mean coefficient of variation for flushing times from the nine initial reservoirs is 2.36) because some particles visit the stable reservoir, which is mobilized infrequently.

The mature forest cover on the stable reservoir attests to its infrequent mobility compared to the more active reservoirs. Mean flushing times for particles starting in the stable reservoir are 2–10 times longer (Tables 3 and 4) than particles starting in any of the other reservoirs. The relative mobility of the stable reservoir profoundly affects the dynamics of all the sediment reservoirs. Initial model runs where the stable reservoir was allowed to interact more frequently with the other reservoirs gave reservoir flushing times that were unreasonably long for the stable reservoir, based on residence time estimates for that reservoir. A minimal interaction of more active particles with those in the stable reservoir gave the most realistic model. Although the 1964 flood is the largest sediment-transporting event of record and caused changes in channel morphology that have lasted for decades, it was not such an infrequent event as to significantly mobilize the stable reservoir. "Minimal" in the case of Redwood Creek therefore means remoblization of the stable reservoir by an event that has a recurrence interval greater than approximately 100 years. This suggests that infrequent, high-intensity flood events are responsible for major long term shifts in sediment storage, and more frequent, moderate intensity flood events, which do not significantly mobilize the stable reservoir, cannot effect such changes.

One application of the model lends insight into the use of individual sediment particles as tracers for measuring sediment mobility. The mean transit time for a particle traveling down the middle channel reach was almost 6 times greater than the median transit time. Sediment tracer studies are therefore limited in that any single particle has a large range of possible transit rates, and the distribution of transit times has a strong position skew. Unless a large number of particles are marked, and most of them recovered, it is difficult to characterize the flux of the reservoir being studied.

The Markov model can graphically portray the passage of a slug of sediment through time in a channel reach, following a brief period of major sediment input. For example, the model for the middle reach of Redwood Creek graphically shows the passage of a slug of sediment through the active reservoir in the 18 years following the 1964 storm. Such movement behavof concentrations of channel-stored sediment has been observed elsewhere [*Gilbert*, 1917; *Mosley*, 1978; *Kelsey*, 1982; *Meade*, 1982; *Madej*, 1982], as well as in Redwood Creek [*Nolan and Janda*, 1979; *Madej*, 1984]. The model suggests the wave form is primarily due to sediment mobilization in the active reservoir and that sediment that visits the semiactive or inactive reservoir will cease to contribute to the downstream migration of the wave form. Temporary storage in the semiactive and inactive reservoirs may well contribute to diffusion of the wave. Probably for this reason, as well as sorting by size [*Mosley*, 1978], sediment waves have increasing wavelengths and decreasing amplitudes in the downstream direction.

Because the model appears to accurately portray the movement of such sediment slugs, it can predict the length of time for a channel to recover from the aggradation that accompanies such a change. In the middle reach of Redwood Creek, the interval of peak flux of stored sediment through the active reservoir was approximately 18 years. The time for stored sediment volumes to decrease to those of pre-1964 conditions will be approximately 30 years. The capability to predict future sediment storage changes in Redwood Creek, given different possible annual discharge scenarios, may be an especially valuable management tool for this basin. The lower third of the basin is part of the Redwood National Park, and the upper two thirds are owned mainly by timber companies who manage the land for logging. Given initial amounts and distribution of stored sediment and the geometry of the channel, modeling sediment storage changes could be used to predict the depth and duration of periodic channel aggradation. Such aggradation may result from erosion by infrequent storms acting alone or in concert with various degrees of logging-induced ground disturbance upstream.

Our evaluation of the method is that a stochastic approach to modeling stored sediment is feasible. Application of the method requires a drainage basin with a good sediment transport data base. Even with such a data base, some of the residence times and reservoir interaction probabilities must be estimated. These estimates can be evaluated on the basis of whether they are consistent or inconsistent with field data. Some of this field data is easy to obtain (ages of vegetation, aerial photographs, channel surveys) and other field data requires measurements over many years (monitoring volume changes). Given these qualifications, a sample application of the model shows that it can provide information which is both useful for practical purposes as well as for gaining a better understanding of processes.

APPENDIX: EXAMPLE OF THE FINITE MARKOV CHAIN ANALYSIS OF RESERVOIR FLUSHING TIMES USING THE FUNDAMENTAL MATRIX N

Step 1

$$Q_{2,\text{mod}} = \begin{bmatrix} \text{transition matrix for} \\ \text{middle reach given} \\ \text{moderate interaction} \\ \text{with stable reservoir} \end{bmatrix}$$

$$= \begin{bmatrix} 0.9091 & 0.0812 & 0.0046 & 0.0045 \\ 0.0278 & 0.9444 & 0.0167 & 0.0111 \\ 0.0061 & 0.0054 & 0.9846 & 0.0039 \\ 0.00016 & 0.00015 & 0.00022 & 0.99947 \end{bmatrix}$$

Step 2

$$N = (I - Q_{2,\text{mod}})^{-1} = \begin{bmatrix} 15.7 & 8.35 & 20.3 & 458 \\ 15.7 & 31..9 & 56.8 & 1{,}220 \\ 15.7 & 19.5 & 110 & 1{,}351 \\ 15.7 & 19.6 & 67.9 & 2{,}931 \end{bmatrix}$$

Step 3

Row summations of jth row of N:

$$j_1 = 502 \text{ years}$$

$$j_2 = 1320 \text{ years}$$

$$j_3 = 1500 \text{ years}$$

$$j_4 = 3030 \text{ years}$$

where j_1 is the active reservoir, j_2 is the semiactive reservoir, j_3 is the inactive reservoir, and j_4 is the stable reservoir (row sum = flushing time; see text).

Acknowledgments. L. Reid and G. Williams, as well as two anonymous reviewers, provided most helpful reviews of this paper. The authors, however, take sole responsibility for any conceptual errors contained herein. The senior author is grateful for support from Redwood National Park during the formative stages of this project.

REFERENCES

Brown, W. M., Sediment transport, turbidity, channel configuration and possible effects of impoundment of the Mad River, Humboldt County, California, *U.S. Geol. Surv. Water Resour. Invest. Pap., 26-75*, 63 pp., 1975.

Dacey, M. F., and A. Lerman, Sediment growth and aging as Markov chains, *J. Geol., 91*, 573–590, 1983.

Dietrich, W. E., and T. Dunne, Sediment budget for a small catchment in mountainous terrain, *Z. Geomorphol., 29*, 191–206, 1978.

Dietrich, W. E., T. Dunne, N. F. Humphrey, and L. M. Reid, Construction of sediment budgets for drainage basins, *Gen. Tech. Rep. PNW-141*, pp. 5–23, U.S. For. Ser., Washington, D. C., 1982.

Gilbert, G. K., Hydraulic-mining debris in the Sierra Nevada, *U.S. Geol. Surv. Prof. Pap., 105*, 154 p., 1917.

Harden, D. R., R. J. Janda, and K. M. Nolan, Mass movement and storms in the drainage basin of Redwood Creek, Humboldt County, California—A progress report, *U.S. Geol. Surv. Open File Rep., 78-486*, 161 pp., 1978.

Helley, E. J., and V. C. LaMarche, Historic flood information for northern California streams from geological and biological evidence, *U.S. Geol. Surv. Prof. Pap., 485-E*, 1973.

Heyman, D. P., and M. J. Sobel, *Stochastic Models in Operations Research*, vol. 1, 548 pp., McGraw-Hill, New York, 1982.

Isaacson, D. L., and R. W. Madsen, *Markov Chains: Theory and Applications*, 256 pp., John Wiley, New York, 1976.

Kelsey, H. M., A sediment budget and an analysis of geomorphic process in the Van Duzen River basin, north coastal California, 1941–1975, *Geol. Soc. Am. Bull., 91*, 1119–1216, 1980.

Kelsey, H. M., Hillslope evolution and sediment movement in a forested headwater basin, Van Duzen River, north coastal California, *Gen. Tech. Rep. PNW-141*, pp. 86–96, U.S. For. Serv., Washington, D. C., 1982.

Kelsey, H. M., Geomorphic processes in the recently uplifted Coast Ranges of Northern California, in *Geomorphic Systems of North America, Decade of North American Geology Centennial Special Volume 2*, Geological Society of America, Boulder, Colo., in press, 1987.

Kelsey, H. M., M. Coghlan, J. Pitlick, and D. Best, Geomorphic analysis of streamside landsliding in the Redwood Creek basin, *U.S. Geol. Surv. Prof. Pap., 1454*, in press, 1987.

Knott, J. M., Sedimentation in the Middle Fork Eel River Basin, California, *U.S. Geol. Surv. Open File Rep.*, 60 pp., 1971.

Lamberson, R. H., An environmental problem, *Coll. Math. J., 8*, 252–253, 1977.

Madej, M. A., Sediment transport and channel changes in an aggrad-

ing stream in the Puget Lowland, Washington, *Gen. Tech. Rep. PNW-141*, pp. 97–108, U.S. For. Serv., Washington, D. C., 1982.
Madej, M. A., Recent changes in channel-stored sediment, Redwood Creek, California, *Tech. Rep., 11*, 54 pp., Redwood Natl. Park Serv., Arcata, Calif., 1984.
Meade, R. H., Sources, sinks, and storage of river sediment in the Atlantic drainage of the United States, *J. Geol., 90*, 235–252, 1982.
Mosley, M. P., Bed material transport in the Tamaki River near Dannevirke, North Island, New Zealand, *N. Z. J. Sci., 21*, 619–626, 1978.
Nolan, K. M., and R. Janda, Recent history of the main channel of Redwood Creek, in *Guidebook for a Field Trip to Observe Natural and Resource-Management Related Erosion in Franciscan Terrane of Northern California*, pp. X-1-X-16, Geological Society of America, Cordilleran Section, Fresno, Calif., 1979.
Trimble, S. W., The fallacy of equilibrium in contemporary denudation studies, *Am. J. Sci., 277*, 876–887, 1977.
Trimble, S. W., Changes in sediment storage in the Coon Creek basin, Driftless Area, Wisconsin, 1853 to 1975, *Science, 214*, 181–183, 1981.
United States Geological Survey, Water Resources Data for California, vol. 1, Pacific Coast Basins, technical reports, Washington, D. C., 1974–1982.
Varnum, N., Channel changes at cross sections in Redwood Creek, California, *Tech. Rep. 12*, 51 pp., Redwood Natl. Park Serv., Arcata, Calif., 1984.
Waananen, A. O., D. D. Harris, and R. C. Williams, Floods of December 1964 and January 1965 in the Far Western States, 1, Description, *U.S. Geol. Surv. Water Supply Pap., 1866-A*, 265 pp., 1971.
Wilkin, D. C., and S. J. Hebel, Erosion, redeposition, and delivery of sediment to midwestern streams, *Water Resour. Res., 18*, 1278–1282, 1982.
Zinke, P. J., Floods, sedimentation, and alluvial soil formation as dynamic processes maintaining superlative Redwood groves, in *Watershed Rehabilitation in Redwood National Park and Other Pacific Coastal Areas*, edited by R. N. Coats, pp. 26–49, John Muir Institute, San Francisco, Calif., 1981.

H. M. Kelsey, Department of Geology, Western Washington University, Bellingham, WA 98225.
R. Lamberson, Department of Mathematics, Humboldt State University, Arcata, CA 95521.
M. A. Madej, Redwood National Park, 1125 16th Street, Arcata, CA 95521.

(Received July 1, 1986;
revised May 19, 1987;
accepted May 21, 1987.)